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The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-32

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OPTIMUM MODULATION AND MULTIPLEXING TECHNIQUES FOR APOLLO SPACECRAFT TO GROUND COMMUNICATIONS

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1. INTRODUCTION

This report describes the work performed by The Bissett-Berman Corporation to establish an optimum modulation and multiplexing technique for voice, telemetry, television, and ranging, as applied to Apollo spacecraft/ground communications. The work was carried out in conjunction with a parallel effort by the Apollo Telecommunications group of North American Aviation. A major objective was to compare digital and analog techniques taking into account availability of advanced coherent (phase lock loop) demodulation for the latter.

The signal parameters assumed for the study are

1.	TV picture	15 frames/sec 500 lines/frame 1 mc bandwidth 34 db peak signal to rms noise ratio
2.	Telemetry	Approximately 100 kilobits/sec 10^{-6} error rate
3.	Voice	3 kc bandwidth Nominal 30 db signal-to-noise ratio

The television picture resolution and frame rate were specified less than commercial standards in order to conserve transmitter power while providing acceptable picture quality. The power required is, of course, affected by the channel parameters: 2300 mc carrier frequency, 54 inch transmitting dish, 85 foot receiving dish, and 300°K receiver noise temperature.

No specific analysis is made in the report of ranging. The JPL pseudonoise ranging technique is suggested; however, the television modulation techniques presented are not intended to be used simultaneous with ranging.

Analog (FM) modulation for television does not provide a coherent narrow band carrier. It is presumed that the DSIF ground antenna can still be made to track the spacecraft.

The study is devoted in large part to modulation techniques. The reason for this emphasis is that selection of a method for multiplexing TV, voice, and telemetry must be preceded by examination and selection of optimum modulation techniques. Once a clear indication of the optimum modulation is reached, choice of multiplexing technique is relatively straightforward.

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2. CONCLUSIONS AND RECOMMENDATIONS

A number of reasonably efficient modulation, detection, and multiplexing techniques are compared in this report. Many other familiar modulation and detection techniques have not been discussed because they are known to be much less efficient when a high output signal-to-noise ratio is desired. These discarded techniques include, in particular, double side band and single side band modulations, which characteristically do not yield an "improvement factor". The promising modulation techniques include (1) digital transmission by PCM, (2) reduced bandwidth digital transmission, and (3) frequency modulation with coherent (phase lock loop) demodulation at the receiver.

Despite the common belief that digital transmission is inherently the most efficient method of communication, FM is found to yield a considerably lower power requirement than uncompressed PCM when a phase lock loop discriminator is employed at the receiver. The apparent reason for this situation is that the modulation techniques are compared on the achieved output signal-to-noise ratio, rather than on information rate. Then, the quantizing noise inherent in PCM and the additional noise caused by even a low rate of transmission errors severely penalize PCM in comparison with an analog modulation.

Bandwidth reduction by redundancy removal based on run length coding of picture edges yields a power requirement for digital TV which is more competitive with FM. The final choice between these two modulation techniques must rest on other considerations, such as simplicity and reliability of the spacecraft equipment and growth potential. Nevertheless, a preliminary evaluation favors FM over compressed digital transmission because

1. The spacecraft equipment does appear simpler
2. The timing rate of the compressed digital TV is about 10 mc, which is near state of the art even for the reduced frame rate and horizontal resolution presumed for the study
3. Above threshold, FM has the virtue of improving the output signal-to-noise ratio linearly with the input, so that advantage is taken of operating conditions which are better than the worst case.

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A disadvantage of FM over compressed digital transmission is a somewhat wider spectral occupancy. These conclusions are supported by Table I, which is a brief comparison of the techniques studied. As an additional note, the digital transmission performance may be improved by about 3 db if orthogonal-type multiple-character coding is employed. This increases modulator complexity somewhat and, more significantly, increases the RF bandwidth by about a factor of 3 (for 5-bit characters). However, again increasing modulator complexity, quadriphase modulation could be used to reduce RF bandwidth by a factor of two with no change in the normalized threshold.

As noted, FM and compressed digital modulation are roughly competitive for television transmission. Multiplexing of telemetry and voice with the television is relatively straightforward with either type of TV modulation. If digital TV is used, the telemetry data, which is already digital, and digitalized (PCM) voice may simply be interleaved (Time-division multiplexing). Sending 3 telemetry bits and one voice bit for each 64 TV bits yields the required telemetry and voice data rates at a 2.48 megabit/sec transmission rate. However, this simple approach has the disadvantage that the telemetry error rate specification sets the threshold. The recommended approach, therefore, is to increase the telemetry redundancy. For example by using a 6-bit error-correcting code to convey 3 telemetry bits (50% redundancy), single errors can be corrected, and the telemetry error rate is negligible at the 10^{-5} TV threshold. The digit rate is increased slightly to 2.6 megabits/sec, and the increase in power required is negligible.

With FM television, the telemetry and voice multiplexing can be either frequency division or time division. Frequency division has the advantage of comparative simplicity. That is, the telemetry is biphase modulated on a subcarrier and the voice frequency modulated on another subcarrier, both subcarriers being above the television cutoff (1 mc). This is feasible because the phase lock loop demodulator must have a 3-db bandwidth much larger than the TV cutoff, in order to maintain phase lock on the FM carrier. A theoretical design which allocates the subcarriers with minimum spacing shows that the carrier power requirement is increased only negligibly by the addition of the two subcarriers.

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Picture Quality	Modulation Type	Normalized Threshold 1 mc	Transmitter Power	Telemetry Multiplexing	Equipment Assessment	Approximate RF Bandwidth
15 frames/sec 500 lines/frame 250 elements/line 34 db S/N	PCM	13.8 db	12 W	Time division	State of the art	20 mc
	Compressed Digital	10.6	5.9	Time division	Complex	5 mc
	FM	7.4	2.8	Frequency or Time division	Simple	10 mc

Table I. Comparison of Modulation Techniques

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Despite the equipment simplicity, frequency division multiplexing is not recommended because of a potential crosstalk problem. The phase lock loop demodulator is inherently nonlinear for non-zero phase errors, and this produces cross-modulation terms. Furthermore, if either the transmitter VCO or the demodulator VCO has a nonlinear characteristic, additional cross-talk will result. It will be noted that minor nonlinear distortion of the TV signal is not otherwise a serious problem, since it merely distorts the gray scale of the picture.

Time division multiplexing of the telemetry data and voice is recommended for use with FM transmission because it overcomes the crosstalk problem mentioned above and, yet, requires only a slight increase in modulator complexity. The telemetry and voice data can, most simply, be inserted between the TV lines during flyback. It will be noted that the number of lines per second is 7500, so that the voice may be transmitted, very simply, directly by PAM with one sample per line. The output signal-to-noise ratio will be the same for both voice and TV. On the other hand, a minimum of 14 telemetry bits must be accumulated for transmission at a speeded up rate in order to achieve an average rate of 100 kilobits/sec. The requisite shift register storage is the primary added complexity. The television bandwidth of 1 mc theoretically allows 2×10^6 pulses per second at baseband; however, even using 10^6 pulses per second, transmission of 16 pulses (14 telemetry bits, one voice sample, and a horizontal sync pulse) would occupy only 12 percent of the line duration. Commercial TV standards allow 18 percent for sync, by way of comparison.

It may be noted that synchronizing the telemetry to the TV line rate is desirable because the necessity for transmitting TV sync pulses above the reference black level can thereby be eliminated. That is, the telemetry may include frame sync information which will specify the start of a TV frame and the location of the horizontal sync pulses to an accuracy of at least 10 μsec. Taking the pessimistic 10 μsec value, the horizontal sync pulse must be unique within this uncertainty, so that the total flyback time is increased to 25 μsec. However, this is still only 18.7% of the line duration, or almost identical to commercial standards. For this reason, this method of accomplishing TV sync is recommended.

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As a final comment, the conventional second-order phase lock loop is recommended as an efficient demodulator for FM. The FM feedback demodulator is inferior when a single pole baseband filter is used. Furthermore, an FM feedback demodulator with any baseband filter can always be replaced by a phase lock loop with an appropriate filter and yielding improved performance. An optimum filter can be found to yield several db reduction in threshold of the phase lock loop, so that a search for a better filter than that conventionally used in the second order loop is recommended. A significant improvement, however, is unlikely to be realized in this way.

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3. COMMUNICATION CHANNEL PROPAGATION ANALYSIS

The information bandwidth which can be accommodated for spacecraft-to-ground communications will be limited by the available transmitter power. In order to maximize the information bandwidth, the modulation and multiplexing technique should be selected on the basis of maximum communications efficiency, which means the minimum required transmitter power to achieve the desired performance.

Comparison of different modulation techniques with a variety of transmitted bandwidths may be facilitated by defining a normalized threshold which equals the ratio of signal power to the noise power measured in twice the information bandwidth. That is, if S is the received signal power at threshold, N_o is the noise spectral density, and f_m is the information bandwidth, the normalized threshold is

$$\text{Normalized Threshold} = \frac{S}{2N_o f_m} \qquad (3-1)$$

The factor of 2 is related to the convention of using double side band modulation as a standard for comparison.

The relation between transmitter power, information bandwidth, and normalized threshold may be graphically presented for the specific Apollo lunar mission. The transmission parameters assumed are as follows:

Free space loss at 2300 mc	=	212.0 db
Spacecraft antenna gain	=	27.0 db min.
Spacecraft feed loss	=	2.0 db max.
Space craft pointing loss	=	1.5 db max.
DSIF 85' dish gain	=	51.5 db min.
Ground feed loss	=	0.1 db max.
Receiver noise density	=	-203.0 dbw/cps max.

The net result is a power of -2.9 dbw (0.51 watt) to produce a normalized threshold of 0 db for an information bandwidth of one megacycle. Figure 3-1 shows the power required versus information bandwidth, with normalized threshold as a parameter. Of course, the final transmitter power specification would still have to include a safety margin to allow for below nominal operation of the spacecraft power amplifier.

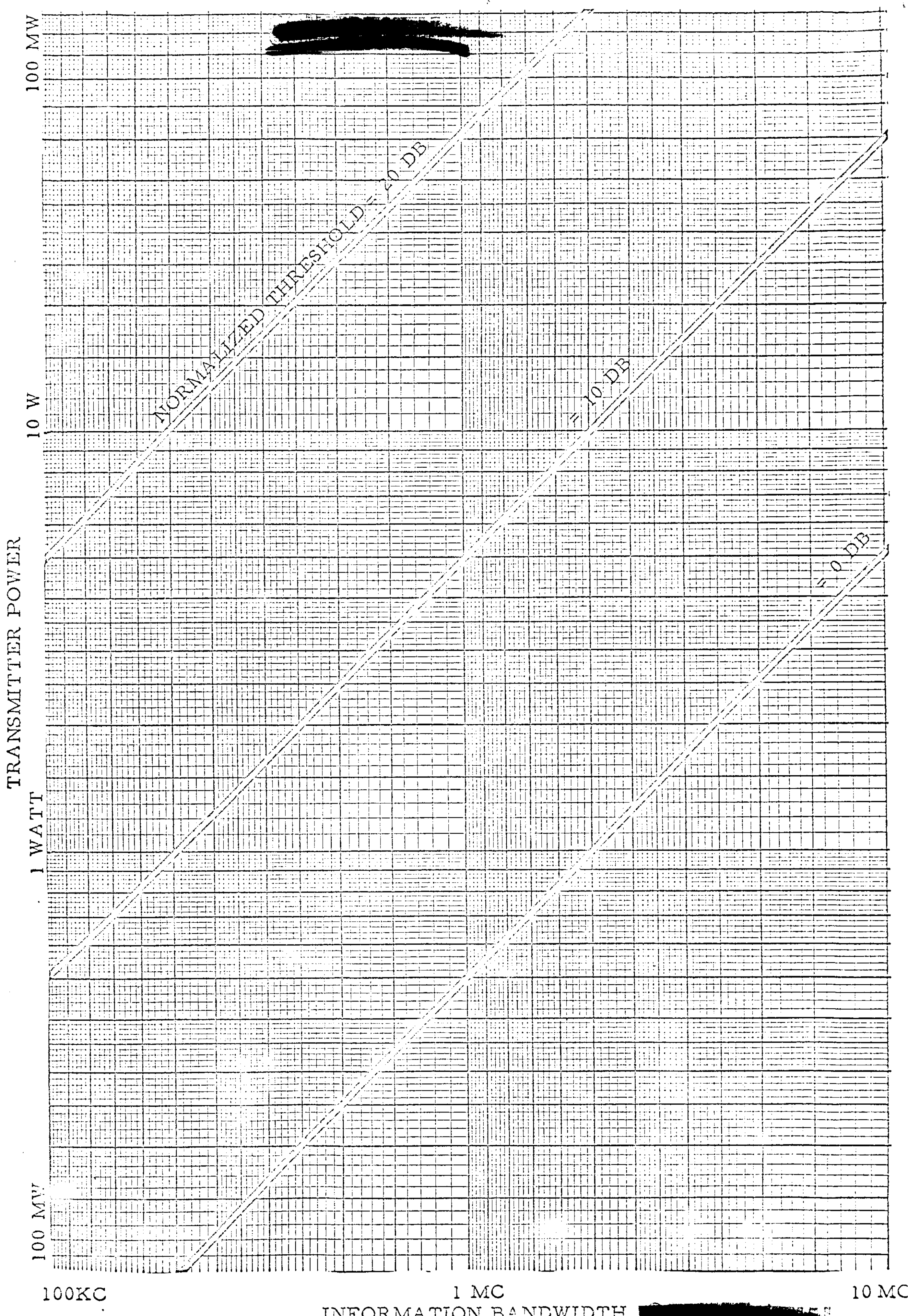

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Figure 3-1. Required Transmitter Power

4. DIGITAL TRANSMISSION OF AN ANALOG SIGNAL

When analog information must be transmitted over a digital channel, the inherent communications efficiency of digital modulation techniques is compromised by the fidelity criterion used to evaluate channel performance. It is well known that coding techniques involving a large alphabet size exist capable of approaching the theoretical Shannon limit. Nevertheless, even if the theoretical limit is fully reached, digital transmission does not yield perfect reproduction of the analog information, because of quantizing noise. Furthermore, transmission errors in a practical digital channel introduce additional noise into the reproduced signal.

For TV pictures, the fidelity criterion ultimately is a subjective evaluation of picture quality. However, the relation between signal-to-noise ratio and picture quality has been empirically determined for Gaussian noise. This relation is presumed to be also valid for noise due to quantization and transmission errors, in order to allow analytical comparison of analog and digital transmission techniques. Actually, a special pseudonoise technique, described later, is required to avoid false contour effects, which ordinarily make quantizing noise more severe than Gaussian noise. Similarly, the effect of transmission errors is more similar to impulse than to Gaussian noise, and these are not equivalent on a power basis.

The analytical technique to be followed consists of evaluating noise due to quantization and transmission errors and adding them on a power basis, since they are independent. The transmission parameters are then adjusted to maximize the resultant output signal-to-noise ratio. Unless further subjective evaluations deem otherwise, the resulting system design is presumed optimum for the modulation type.

4.1 SAMPLING AND QUANTIZING

A band-limited low-pass signal (maximum frequency = f_m) ideally can be represented with sample points spaced by the reciprocal of twice the bandwidth (Nyquist rate = $2f_m$). Each sample will be quantized into one of L levels, which corresponds to $J = \log_2 L$ binary digits. Thus, the binary digital rate required to convey the analog signal is $2f_m \log_2 L$.

For purposes of comparison with other modulation techniques, the signal is presumed to have a zero average value, and the quantum levels are taken to be uniformly spaced. The levels for each polarity range from $Q/2$ to $(L-1)Q/2$, where L is even and Q is the quantum step.

If the various levels are equally probable, the average output signal power is computed[1] to be

$$S = \sum_{k=1}^{L/2} \left(k - \frac{1}{2}\right)^2 Q^2 = \frac{Q^2}{12}\left(L^2 - 1\right) \tag{4-1}$$

That is, the average power is, very closely, one third of the peak power.

4.2 QUANTIZING NOISE AND TRANSMISSION ERRORS

The usual assumption for quantizing noise is that the analog signal is uniformly distributed over a quantum step.[2] We shall consider only uniform spacing of the quantum steps here. If the step width is Q, the mean square value of the quantizing noise is

$$N_Q = \frac{1}{Q}\int_{-Q/2}^{Q/2} n^2\, dn = \frac{Q^2}{12} \tag{4-2}$$

The mean value is, of course, zero.

The effect of transmission errors is to cause some samples to be received on a incorrect level. If the error corresponds to a displacement of D levels, the amplitude error is DQ. The total mean square error in the output signal may then be expressed as

$$N = N_Q + \overline{(DQ)^2} \tag{4-3}$$

where the displacement if presumed to be zero on the average. The output signal-to-noise ratio is

$$\left(\frac{S}{N}\right)_{out} = \frac{(L^2 - 1)/12}{\frac{1}{12} + \overline{D^2}} \tag{4-4}$$

where $\overline{D^2}$ remains to be evaluated for particular digital transmission techniques. Increasing the number of levels decreases quantizing noise but increases the probability of transmission errors. An optimization is desired for particular transmission techniques.

4.3 ROBERTS' PSEUDONOISE TECHNIQUE

The above analysis of quantizing noise is not directly applicable to PCM transmission of television pictures because of the false contours introduced by the amplitude quantization. That is, in a region of slowly changing intensity, there must be sharp discontinuity in the reproduced picture whenever the intensity crosses from one quantum level to the next. To eliminate annoyance from these false contours, 6-bit quantization is normally required.

Roberts has devised a pseudonoise technique to eliminate this problem.(3) Basically, noise, uniformly distributed over a quantum step, is added to the video signal before quantizing. The noise is actually pseudorandom, so that it can be duplicated at the receiver. The predictable noise is subtracted from the video signal after digital-to-analog conversion in the receiver, with the result that the quantizing noise model previously presumed becomes an exact representation. In other words, Roberts' modulation produces an output signal-to-noise ratio exactly as computed in equation 4-4.

In the absence of transmission errors, equation 4-4 yields an output signal-to-noise ratio of 18 db (28.8 db peak) for 3-bit quantization and 24 db (34.8 db peak) for 4-bit quantization. According to subjective picture quality evaluations presented in a later section, these signal-to-noise ratios produce, respectively, "acceptable" and "fine" pictures. This general conclusion was also reached by Roberts after examination of TV pictures subjected to pseudonoise quantizing at, respectively, 3 and 4 bits. The performance curves presented in the following are based on equation 4-4 and, therefore, inherently presume use of Roberts' pseudonoise technique. Direct PCM would actually require a greater transmitter power because 6 bits would be needed to eliminate false contours.

4.4 BIPHASE MODULATION

With biphase modulation (the most efficient binary modulation), the samples of the analog signal are conveyed by J separate binary digits, each of which has a probability of error P_e. The k^{th} level may be related to the binary digits by the expansion

$$k = -\frac{L-1}{2} + \sum_{j=0}^{J-1} a_j^{(k)} 2^j \tag{4-5}$$

where $a_j^{(k)}$ is j^{th} binary digit in the expansion of the number k. This allocation of levels may not yield the minimum disturbance due to transmission errors, but is simply decoded by exponential weighting of the digits. Furthermore, the effect of transmission errors is easily computed, as will now be seen.

The number of levels displacement due to a transmission error may be expressed as

$$D = \sum_{j=0}^{J-1} \left[a_j^{(T)} - a_j^{(R)}\right] 2^j \tag{4-6}$$

where $a_j^{(T)}$ is the transmitted digit and $a_j^{(R)}$ is the possibly erroneous received digit. Now, if all quantum levels are equally likely, $a_j^{(T)}$ is equally likely to be 1 or 0. Hence,

$$\left.\begin{aligned} a_j^{(T)} - a_j^{(R)} &= 0 \text{ with probability } 1 - P_e \\ &= 1 \text{ with probability } P_e/2 \\ &= -1 \text{ with probability } P_e/2 \end{aligned}\right\} \tag{4-7}$$

so that its mean value is zero. Then $\overline{D} = 0$, as required previously, and

$$\begin{aligned} \overline{D^2} &= \sum_{j=0}^{J-1} \sum_{\ell=0}^{J-1} \overline{\left[a_j^{(T)} - a_j^{(R)}\right] \left[a_\ell^{(T)} - a_\ell^{(R)}\right]} \; 2^{j+\ell} \\ &= \sum_{j=0}^{J-1} \overline{\left[a_j^{(T)} - a_j^{(R)}\right]^2} \; 2^{2j} = P_e \sum_{j=0}^{J-1} 2^{2j} \end{aligned}$$

$$= \frac{P_e}{3} (2^{2J} - 1) = \frac{P_e}{3} (L^2 - 1) \tag{4-8}$$

where $L = 2^J$. (See also Appendix I of reference 1.)

The output signal-to-noise ratio is given from equation 4-4 to be

$$\left(\frac{S}{N}\right)_{out} = \frac{(L^2 - 1)/12}{\frac{1}{12} + \frac{P_e}{3} (L^2 - 1)} = \frac{1}{\frac{1}{(L^2-1)} + 4P_e} \tag{4-9}$$

To maximize this by choice of L requires relating P_e to L. Now, the J digits are transmitted in the sample interval, $1/2 f_m$; hence, the transmission time per digit is $1/2f_m J$. Letting the average signal power be S and the (one-sided) noise spectral density be N_o, we may express the signal-to-noise ratio referred to the RF bandwidth $2f_m$ as $S/2N_o f_m$; this is the normalized threshold. Noting that the RF noise bandwidth corresponding to integration over the time T is 1/T, the expression for error rate is found(4) to be

$$P_e = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{S/N_o f_m J}} e^{-x^2/2} dx \tag{4-10}$$

which incorporates the normalized signal-to-noise ratio defined above. Equation 4-10 is based on availability of a coherent reference at the receiver for optimal demodulation.

Maximization of equation 4-9 for a given normalized power is best carried out numerically, since interest is limited to integral values of J. Calculation shows that the output signal-to-noise ratio is maximized usually by choosing J equal to the minimum integer for which the quantizing noise is sufficiently small. Although increasing J over this minimum value allows a greater P_e, this does not compensate for the greater noise bandwidth. The results are presented in Table II and figure 4-1.

$(S/N)_{out}$	$S/2N_o f_m$	No. of Digits (J)	Error Rate
9.2 db	6.9 db	2	10^{-2}
19.2	12.2	4	2×10^{-3}
29.2	15.7	6	2×10^{-4}
39.2	17.8	7	10^{-5}
49.2	19.8	9	2×10^{-6}

Table II Biphase PCM

The curve in figure 4-1 is not accurate in the sense that it has been drawn smoothly through a small number of computed points. Actually, the optimum number of bits per sample will be constant over a small range of signal-to-noise ratio. Over this range, the relation between output S/N and normalized threshold is determined entirely by the error rate expression, since the quantizing noise is fixed. The points at which the curves for different J cross indicate where the number of bits per sample should be changed. These curves, based on equations 4-9 and 4-10, are presented in figure 4-2. However, as can be seen, the assumption of a smooth curve, as made in figure 4-1, is actually quite reasonable.

The bandwidth occupied by the modulated signal depends on the digit rate. Although the noise bandwidth of the matched filter equals the digit rate, there are spectral components further spread out. A safe assumption is that the bandwidth need not exceed twice the digit rate.

4.5 REGULAR SIMPLEX CODING

A more efficient method for transmitting digital information is to employ a nonbinary alphabet. Orthogonal and biorthogonal coding represent examples of this type of coding.[5] However, the most efficient is regular simplex coding, whereby the equal-energy waveforms have the least correlation (-1/L, where L is the number of waveforms). The error rate performance of regular simplex codes may be obtained directly from that of orthogonal codes by applying the correction factor (L-1)/L to the signal power computed for the orthogonal codes.

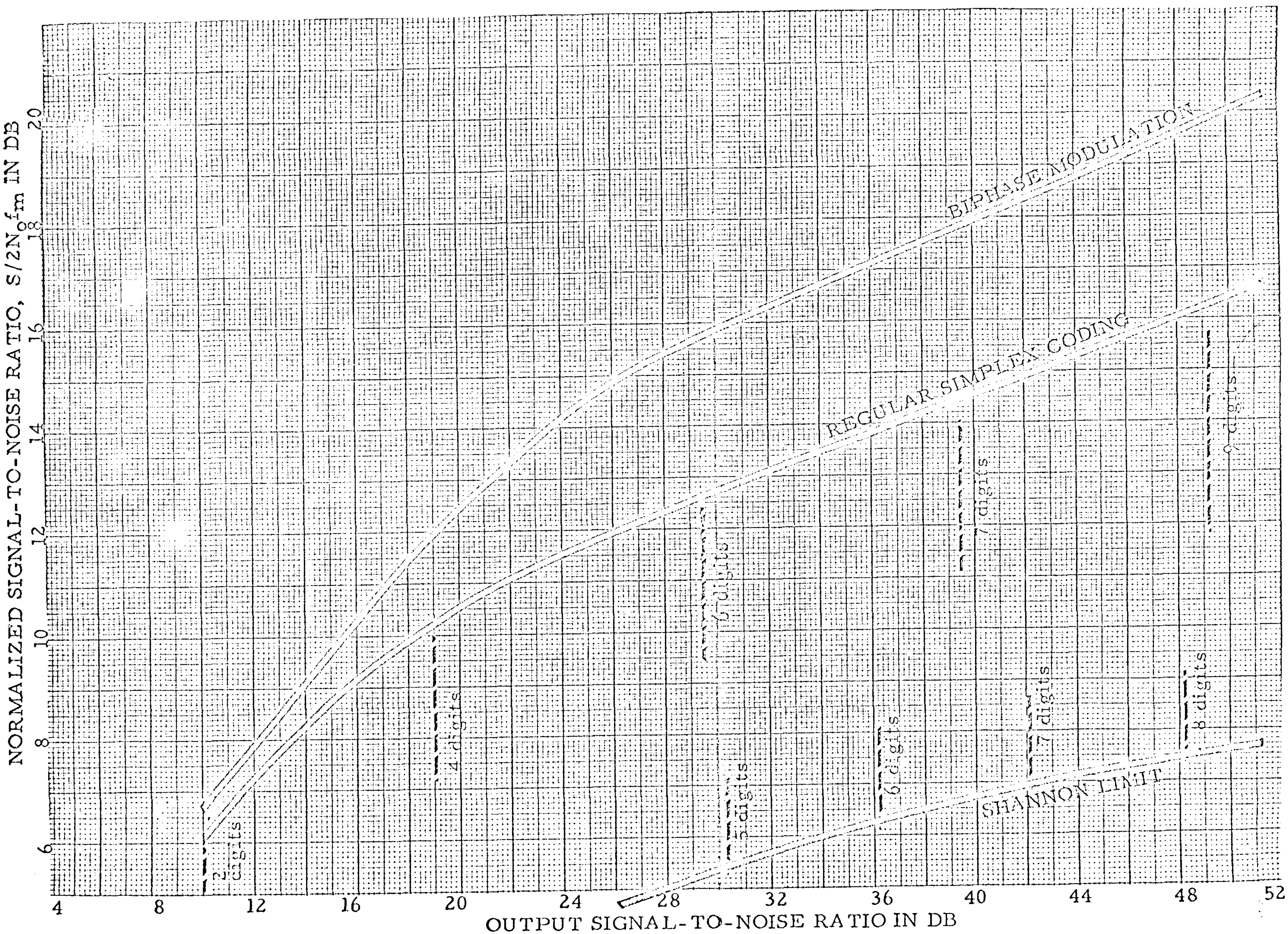

Figure 4-1. Performance of Digital Transmission Techniques

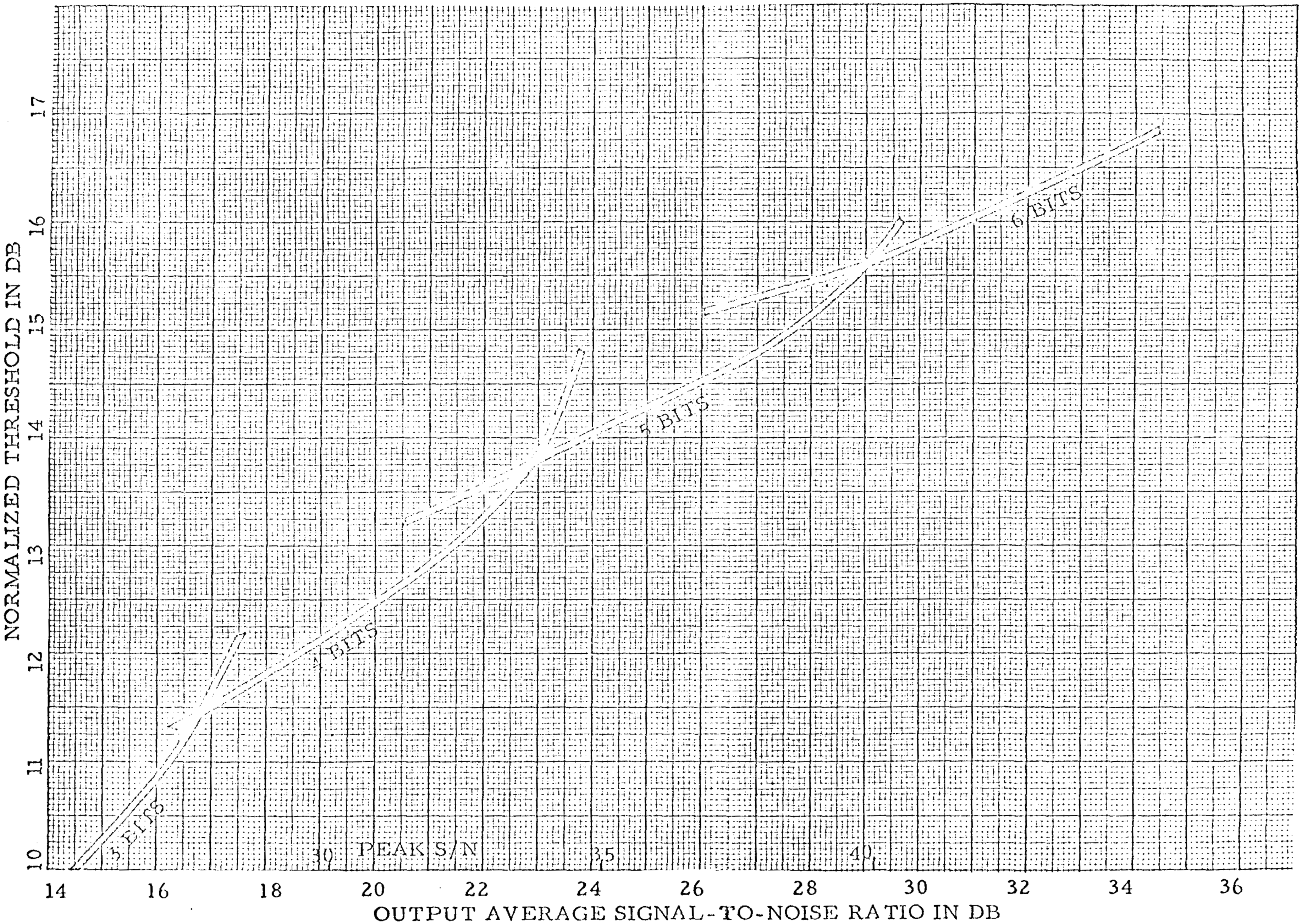

Figure 4-2. Precise Biphase Performance

Regular simplex coding has the property that the demodulation process is equally likely to give any of the L-1 erroneous waveforms, when an error occurs. Thus, the computation of the effect of transmissions errors on signal-to-noise ratio is easily accomplished. It is easy to see that the average number of levels displaced is zero under these circumstances.

In the present discussion, we shall assume that each amplitude level is represented by one of the regular simplex waveforms. Thus, L is both the number of quantum levels and the number of waveforms. A more complicated coding might take two or more samples together, but this is not considered here.

Letting $k^{(T)}$ denote the transmitted level and $k^{(R)}$ the possibly erroneous received level, the displacement is

$$D = k^{(T)} - k^{(R)} \tag{4-11}$$

Since $k^{(R)} = k^{(T)}$ with probability $1 - P_e$ and any other level with probability $P_e/(L-1)$, the average displacement is

$$\overline{D} = \frac{P_e}{L-1} \frac{1}{L} \sum_{1}^{L} \sum_{1}^{L} \left[k^{(T)} - k^{(R)} \right] = 0 \tag{4-12}$$

as stated above. The mean square displacement is

$$\begin{aligned} \overline{D^2} &= \frac{P_e}{L-1} \frac{1}{L} \sum_{1}^{L} \sum_{1}^{L} \left[k^{(T)} - k^{(R)} \right]^2 \\ &= \frac{P_e}{6} L(L+1) \end{aligned} \tag{4-13}$$

(See also appendix I of reference 1.)

Hence, from equation 4-4, the output signal-to-noise ratio is

$$\left(\frac{S}{N}\right)_{out} = \frac{(L^2-1)/12}{\frac{1}{12} + \frac{P_e}{6} L(L+1)} = \frac{1}{\frac{1}{(L^2-1)} + \frac{2P_e L}{(L-1)}} \tag{4-14}$$

Again, the optimization of output signal-to-noise ratio depends on relating P_e to J, using curves derived for orthogonal code signalling with coherent demodulation.[5] These curves have an abscissa which is the signal energy per bit divided by the noise power per cycle. Since the bit duration is $1/2f_m J$ and the (one-sided) noise spectral density is N_o, this abscissa is

$$\frac{S/2f_m J}{N_o} = \frac{1}{J}\left(\frac{S}{2N_o f_m}\right) \tag{4-15}$$

where the quantity in parentheses is the normalized threshold previously defined. For simplicity, the optimization has been carried out for integral J, although L is not truly restricted to being a power of 2. The results are presented in Table III and figure 4-1. Curves similar to those for biphase modulation in figure 4-2 have not been computed.

$(S/N)_{out}$	$S/2N_o f_m$	No. of digits (J)	Error Rate
9.2 db	6.5 db	2	2×10^{-2}
19.2	10.4	4	4×10^{-3}
29.2	12.7	6	4×10^{-4}
39.2	14.0	7	3×10^{-5}
49.2	15.3	9	4×10^{-6}

TABLE III, Regular Simplex Coding

Comparison of the results shows that a relatively small improvement is possible with higher level alphabets. Furthermore, the spectral occupancy is greater by the ratio $(2^J - 1)/J$.

If biorthogonal coding is employed instead of regular simplex, the communications efficiency is slightly poorer for J small. For J large, orthogonal, biorthogonal and regular simplex coding merge, performance-wise. Furthermore, biorthogonal coding requires about half the bandwidth, the occupancy ratio being $2^{J-1}/J$.

4.6 THEORETICAL SHANNON LIMIT

We can determine the theoretical bound on the performance of digital systems from Shannon's well known formula[6]

$$R = B \log_2(1 + \frac{S}{N_o B}) \longrightarrow 1.44\ S/N_o \qquad (4\text{-}16)$$

where white Gaussian noise is presumed and the channel bandwidth B is made very large to maximize R. A theorem of information theory states that the limiting power, as specified by equation 4-16, can be approached by coding with as small an error rate as desired. Thus, assuming ideal coding, only quantizing noise remains, so that the output signal-to-noise ratio is simply

$$\left(\frac{S}{N}\right)_{out} = (L^2 - 1) = 4^J - 1 \qquad (4\text{-}17)$$

from equation 4-4, when J bits per sample are transmitted.

Now, since sampling at the rate $2f_m$ is assumed, the information rate is $R = 2f_m J$. The normalized threshold is determined from equation 4-16 to be

$$\left(\frac{S}{2N_o f_m}\right)_{Thresh} = J \frac{S}{N_o R} = 0.693J \qquad (4\text{-}18)$$

Equations 4-17 and 4-18 specify the performance of an ideal digital system, and the corresponding curve is given in figure 4-1.

It may be observed that there is a large separation between the limiting curve and those of practical systems, this separation being more than is usually attributed to coherent digital systems. A qualitative reason for this is the relatively large output noise contribution of even a low error rate, although such error rates introduce a negligible reduction in information rate from the error-free value. Thus, practical digital systems are more severely penalized when output signal-to-noise ratio rather than information rate is used to evaluate system performance.

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5. COMPRESSED DIGITAL TV TRANSMISSION

One of the ways that the required transmitter power can be decreased is by reducing the information bandwidth via redundancy removal. This is theoretically possible for a TV signal because of the picture correlation within a frame and certain characteristics of the human observer. One such technique has been devised by Schreiber[7] and may be termed a method of "synthetic highs". With this approach, two versions of the pictorial information are transmitted: (1) A low-pass filtered signal which reproduces the gray scale over large areas and (2) an edge indicating signal which reproduces the position of the contours accurately but quantizes the amplitude of an edge to a small number of levels. The bandwidth of the TV signal is compressed by this technique because the number of edges in a frame is reasonably small, typically.

To obtain an estimate of the power saving that is realizable, the design of a digital modulator will be presented for the particular parameters of interest; namely, 15 frames/sec, 500 lines/frame, and one megacycle bandwidth. These numbers yield a horizontal resolution corresponding to 267 picture elements per line. However, to fit digital devices, 512 lines/frame and 256 elements/line are actually assumed.

An analysis of the potential compression has been carried out for a 512 by 512 picture by the Raytheon Co., who also presented a tentative design for the spacecraft equipment.[8] However, this design is predicated on slow-scan TV and transmits at a 30 kilobit/sec rate. Therefore, this design is modified both for a reduced horizontal resolution and for an increased frame rate, but the basic compression concept is unchanged. The use of an "electrostatic vidicon", which is necessary in implementing the digitally controlled scan, is presumed feasible at the higher rates.

The Raytheon proposal calls for filtering the lows signal to 1/16 of the original video bandwidth and quantizing amplitude to 4 bits, which yields an adequate 35 db peak signal-to-rms noise ratio at a low error rate. For the highs signal, the position of each edge is specified by 4 bits and the edge amplitude is quantized to 3 bits. Then including an additional bit to aid system operation, each edge is conveyed by 8 bits

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of information. The key item in estimating the expected compression is the average number of edges per line. For a full resolution picture experimental data suggest the number is not greater than 50. For the reduced resolution picture (256 elements/line), the average number of edges will be presumed not greater than 30. The actual number must be measured by further experiment.

The total number of bits to represent a single TV frame may be computed from the above numbers.

$$\text{No. of bits/frame in lows} = \frac{512 \times 256 \times 4}{16} = 32,800$$

$$\text{No. of bits/frame in highs} = 512 \times 30 \times 8 = \underline{122,900}$$

$$\text{No. of bits/frame} = 155,700$$

This means that the required digital transmission rate for 15 frames/sec is 2.34 megabits/sec. Compared with direct PCM transmission with 5 bits per sample, a compression by the factor 4.3 has been realized. However, a power saving by this factor (6.3 db) is not correspondingly implied, since an error rate with PCM of 10^{-3} still yields an output S/N of 33 db. With the transmission of edge positions, on the other hand, a lower error rate is required, since an error will tend to displace the edge from its time location. It is estimated that an error rate of 10^{-5}, corresponding to a small number of errors per frame, is the maximum tolerable. The additional power needed to decrease the error rate from 10^{-3} to 10^{-5} is about 2.8 db when biphase modulation of the carrier is used to transmit the binary digits, giving a net advantage of about 3.5 db to the compressed digital picture.

The implementation of the digital compression technique hinges on a method for averaging the edge information to allow transmission over the channel at a uniform rate. The method proposed by Raytheon is to start and stop the scan under digital control, so that the vidicon itself serves as the buffer storage. The basic scan time interval between successive picture elements must be short enough to allow two scans per frame, allowing both the lows signal and the highs signal to be transmitted. Furthermore, the information describing a given edge must be available as soon as the information describing the preceding edge has been sent. Raytheon has

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specified a scan rate such that 32 basic scan intervals correspond to the time to transmit the 8 bits describing the previous edge. This accommodates 16 element maximum edge spacing, 8 time intervals for 3-bit analog-to-digital conversion, and a safety margin. Since the digit rate is 2.34 megabits/sec, the scan rate is computed to be 9.36×10^6 picture elements per second, and this is the required digital timing rate. The ratio of 4 also corresponds to filtering the lows picture to 1/16 the normal resolution and quantizing to 4 bits.

A block diagram of the required spacecraft equipment is presented in figure 5-1. Two counters are used to derive the horizontal and vertical scan voltages and hold a given picture element whenever the scan is stopped. The general operation is described in the following paragraphs.

To begin with, the lows information is transmitted. The frame is scanned at a uniform rate of 9.36×10^6 elements per second, and the vidicon output is low-pass filtered to 1/16 the corresponding bandwidth, or 292 kc. A uniformly distributed pseudonoise is shown added to the lows signal to eliminate spurious contours, as described in section 4-3 and the resultant is converted to a 4-bit number. A single lows frame occupies 0.014 second.

At the end of the lows scan, the vertical counter output sets up the system to detect edges. The picture is again scanned; however, whenever an edge is detected, the scan is stopped. The edge detector is, essentially, a differentiator and threshold. The edge amplitude is quantized to 3 bits and the edge position is specified by 4 bits from the horizontal counter. This information is inserted into the shift register as soon as the previous contents have been transmitted, and the scan is restarted. With an average of 30 edges per line, the highs scan will be completed in 0.0528 second. However, if there are fewer edges, the average frame rate increases, while with greater picture detail, the frame rate decreases below the normal 15 per second.

Although not of direct concern, mention may be made of the functions required in the ground demodulator. Basically, the ground receiver must store the first scan for subsequent combining with the second scan of each frame. Then, as the lows signal is read out of storage, synthetic edges

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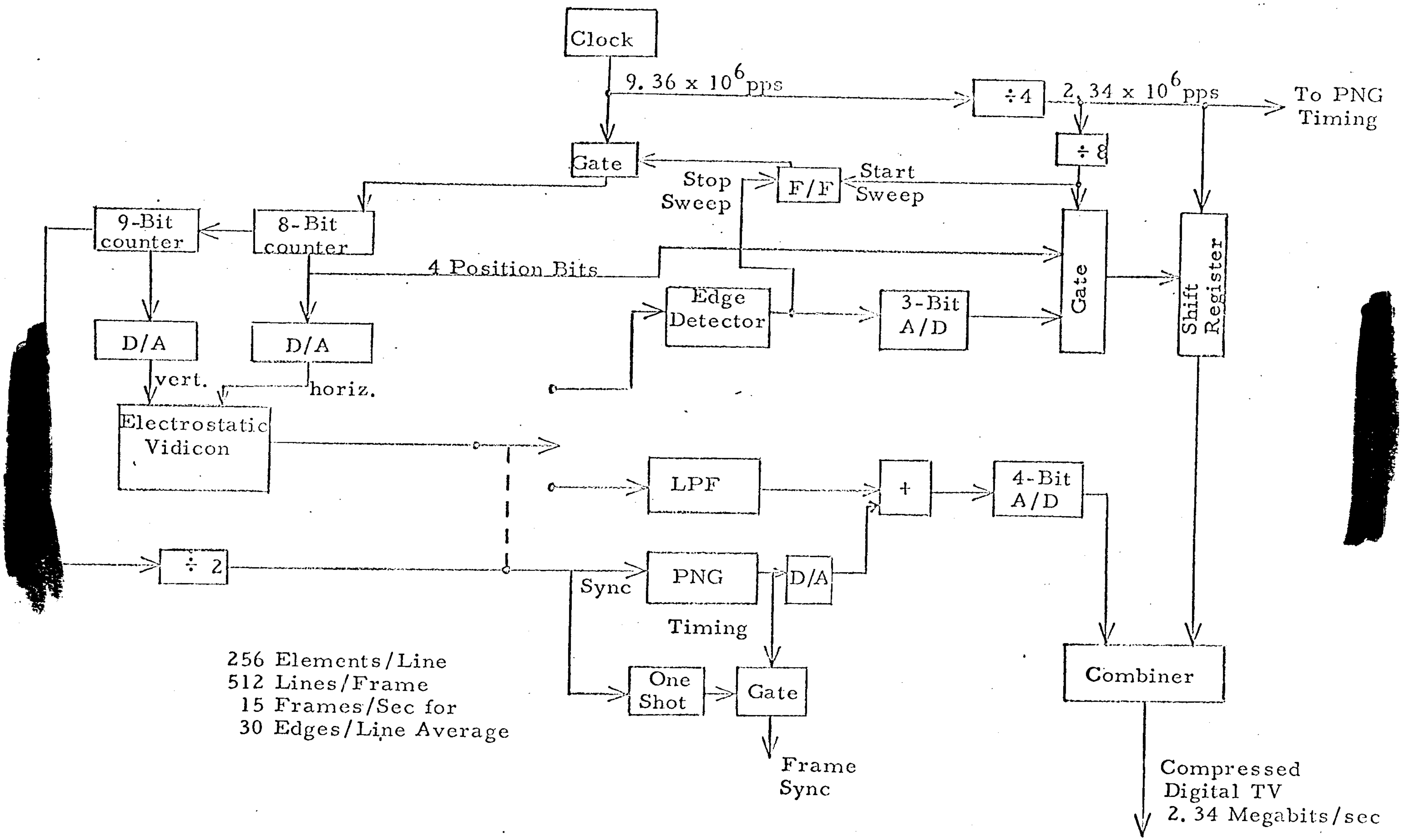

Figure 5-1. Synthetic Highs Digital TV Compression

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are generated from the highs signal with the requisite positions and amplitudes. The synthetic highs signal is combined with the lows signal to reproduce the original picture.

It may be noted that the pseudonoise generator can be synchronized with the TV frame and serves as convenient synchronizing information denoting the start of each frame. The inherent information in the highs signal insures that sync is maintained despite the non-uniform scan during highs transmission.

5.1 MULTIPLEXING OF TELEMETRY AND VOICE WITH COMPRESSED DIGITAL TV

One advantage of digital TV transmission is the feasibility of non interfering multiplexing of the digital telemetry and digitalized voice by interleaving digits. Assuming PCM voice at 32 kilobits/sec (8000 samples/sec, 4 bits/sample), the voice and telemetry data rate is 132 kilobits/sec. Interleaving according to 3 telemetry bits and one voice bit for 64 television bits will approximately yield the desired rates, actually 110 kilobits/sec for telemetry and 36.5 for voice. However, a disadvantage of this direct interleaving is that the required transmitter power is set by the telemetry error rate specification of 10^{-6}. This can be remedied by applying error correcting coding to the telemetry, at the expense of an increased digit rate, of course.

The selected ratio means that the 4 telemetry and voice bits may be inserted after a sequence of 8 edges have been transmitted. However, the actual location of these bits is unimportant as long as the receiver knows when they will occur. Minimum storage is required in the telemetry and voice channels with this interleaving, and demultiplexing may be accomplished by counting 64 TV digits and then stopping the timing pulses supplied to the TV modulator for a duration equal to 4 digits. While the TV modulator is stopped, the telemetry and voice digits are transmitted.

Because the clock is stopped periodically, the clock rate must be increased to 9.9×10^6 pps, and the digit transmission rate becomes 2.48 megabits/sec. The general method of multiplexing is indicated in Figure 5-2. Taking an error rate of 10^{-5} and biphase modulation of the carrier, the required signal-to-noise ratio in a bandwidth of 2.48 megacycles is computed to be 9.6 db. Although this does not directly yield the required

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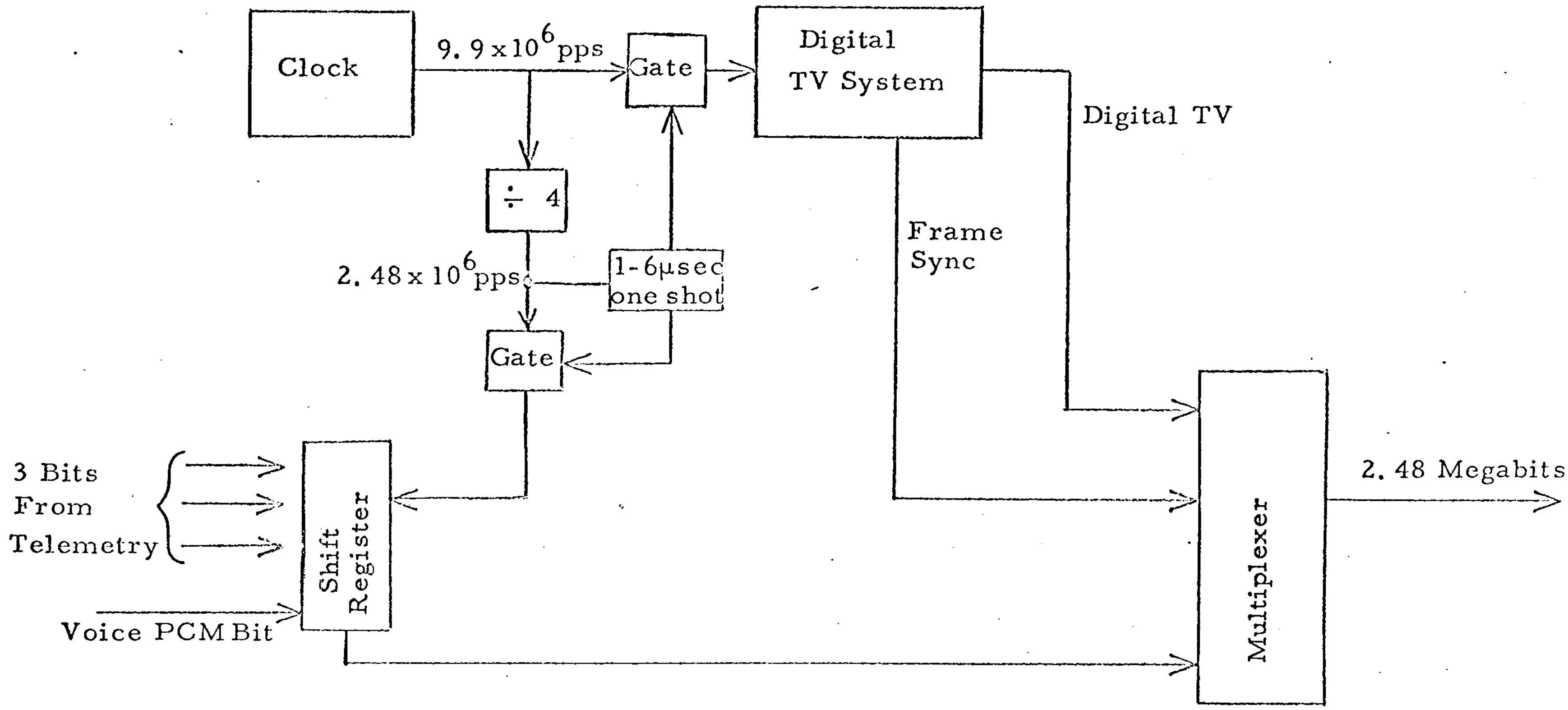

Figure 5-2. Digital Multiplexer.

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telemetry error rate of 10^{-6}, it will allow proper comparison with other modulation techniques. The normalized threshold for the 1 mc video bandwidth (f_m = one megacycle) is then 10.6 db.

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6. ANALOG SIGNAL TRANSMISSION

When an analog signal is to be transmitted at high quality, the modulation techniques employed should reflect this requirement by providing a signal-to-noise ratio improvement factor. Digital transmission accomplishes this objective, as is well known, within limitations imposed by quantization and transmission errors. However, an improvement factor can be realized with analog modulation techniques, also.

In evaluating analog modulation techniques, white Gaussian noise is presumed at the receiver input. As a result the output noise is of a fluctuation nature, and output signal-to-noise ratio is quite reasonable as a fidelity criterion. Thus, the optimum modulation is that which yields the desired output signal-to-noise ratio at the lowest required power. As described in section 3, a convenient way of defining required power is by the normalized threshold; e.g., the ratio of signal power to noise power in the two-sided information bandwidth, equation 3-1.

6.1 CONVENTIONAL FM DISCRIMINATOR

Frequency modulation (FM) is well known to exchange bandwidth for S/N improvement; however, in the past, FM has suffered the penalty of a relatively high threshold. This high threshold is the result of using a noncoherent frequency discriminator in the receiver. The threshold, incidentally, is usually defined as the point at which the linear relation between input and output signal-to-noise ratios breaks down. Thus, below threshold, the S/N improvement is reduced or even negated.

Above threshold, the performance of FM is well known[2] to be given by the relation

$$\left(\frac{S}{N}\right)_{out} = 3\left(\frac{\Delta F}{f_m}\right)^2 \frac{S}{2N_o f_m} \qquad (6\text{-}1)$$

where ΔF is the peak frequency deviation for sine wave modulation at the frequency f_m, and the output filter is presumed to have a sharp cutoff above f_m. Equation 6-1 is easily derived by noting that the output signal from the discriminator has a peak amplitude $2\pi\Delta F$ and the output noise has spectral density $(2\pi f)^2 N_o/S$. This "parabolic noise spectrum" results from the differentiation of phase effectively performed by the discriminator.

Calculations of the actual output noise spectrum have been carried out by Rice[9], and his results have been used by Skinner[10] to derive the relation between input and output signal-to-noise ratios. By observing the points at which the linear relation terminates, the threshold of the discriminator is obtained. The threshold depends on the ratio of predetection and post detection bandwidths, as given in Figure 6-1. An analytical approximation is given by the points marked, according to the equation

$$\left(\frac{S}{N}\right)_{thresh} = 3.8\,(B_{IF}/2f_m)^{0.72} \tag{6-2}$$

The required predetection bandwidth is usually taken approximately equal to the peak-to-peak frequency derivation plus twice the base bandwidth. That is,

$$B_{IF} \cong \left(\frac{\Delta f}{f_m} + 1\right) 2f_m \tag{6-3}$$

Combining equations 6-2 and 6-3 yields

$$\left(\frac{S}{2N_o f_m}\right)_{thresh} = 3.8\left(\frac{\Delta F}{f_m} + 1\right)^{1.72} \tag{6.4}$$

and the corresponding output signal-to-noise ratio is obtained from equation 6-1. Equation 6-4 is plotted in figure 9-2, along with other curves to be derived later.

6.2 PHASE LOCK LOOP DISCRIMINATOR

A phase lock loop can function as an FM discriminator because of its capability for tracking the instantaneous frequency of the modulated carrier. The device frequency deviates a VCO to maintain phase lock and, therefore, to follow the input frequency. The ultimate performance limitation on the phase lock loop is that an excessive phase error drives the loop into a nonlinear region and, very likely, causes loss of lock. This limitation leads to a threshold for the loop dependent on the characteristics of the applied modulation and the loop filter. Fortunately, an analysis to determine loop threshold is much aided by the fact that the phase lock loop is a linear feedback device when the phase error remains small, so that superposition applies.

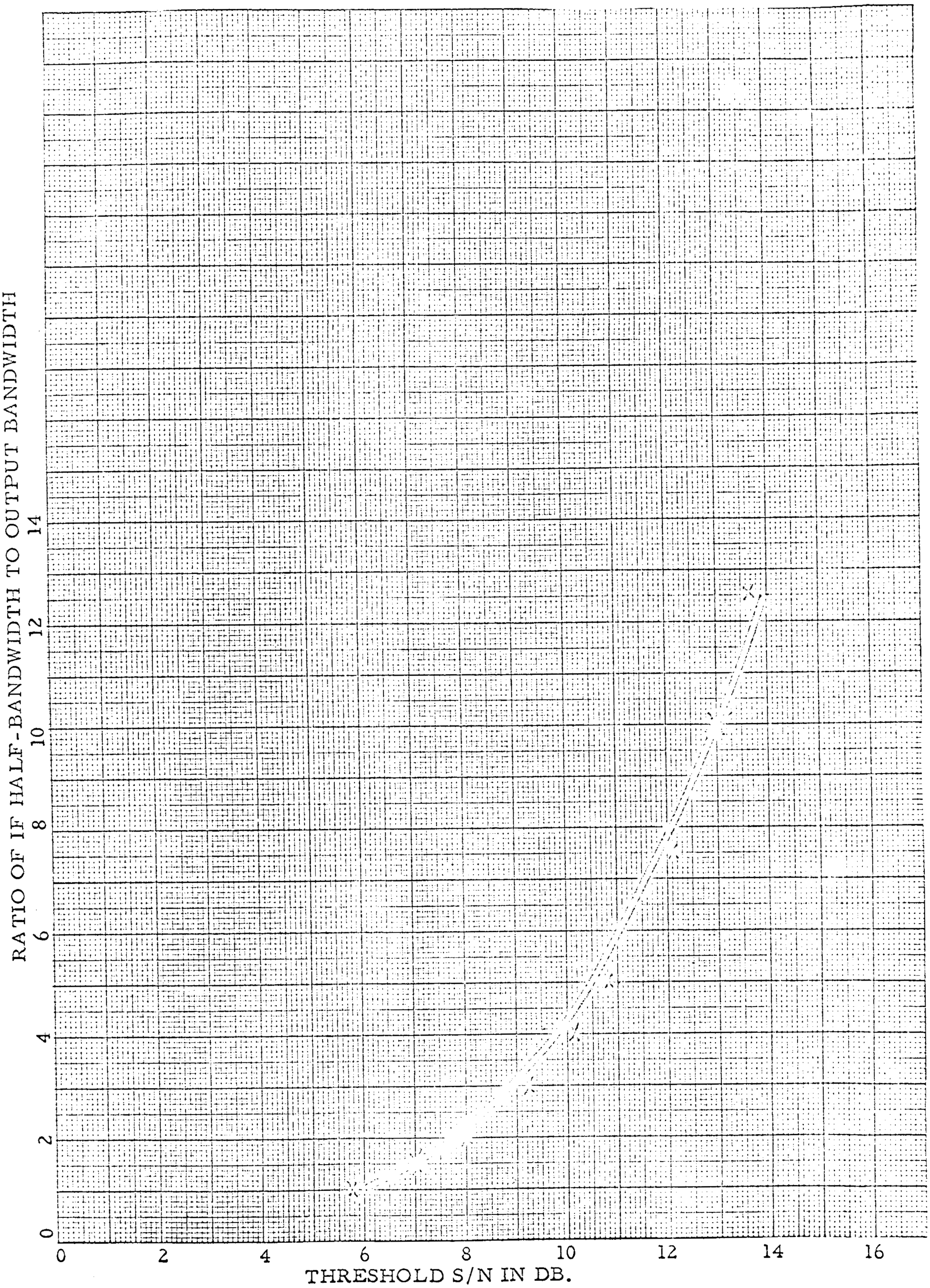

Figure 6-1. Frequency Detector Threshold

To begin, the loop parameters and equations are reviewed. The block diagram of the loop is shown in figure 6-2. The low-pass filter in the output eliminates the output noise outside of the modulation bandwidth but does not affect tracking or change the loop threshold. The input phase modulation is described by the phase θ_i and the output estimate of the frequency modulation is given by the time derivative $s\,\theta_o$.

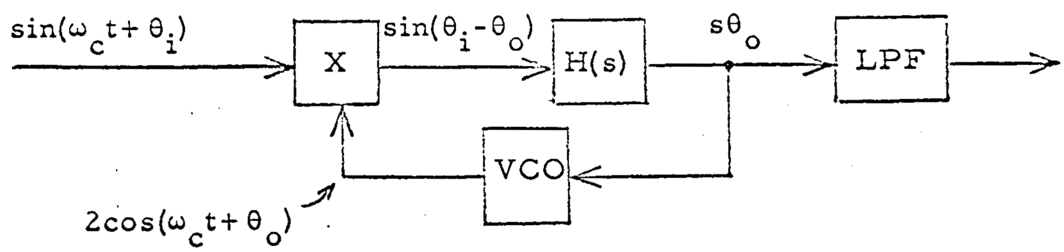

Figure 6-2. Phase Lock Loop

The linearized equivalent for $\theta_i - \theta_o$ small is given in figure 6-3.

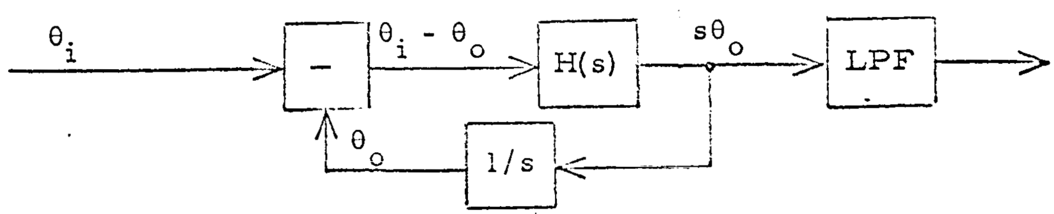

Figure 6-3. Linearized Equivalent Circuit

The loop transfer function for phase or frequency is seen to be

$$\frac{\theta_o}{\theta_i} = \frac{s\theta_o}{s\theta_i} = \frac{H(s)}{s + H(s)} = F(s) \qquad (6\text{-}5)$$

and the transfer function for phase error is

$$\frac{\theta_i - \theta_o}{\theta_i} = \frac{s}{s + H(s)} = 1 - F(s) \qquad (6\text{-}6)$$

The open loop gain is, of course, $H(s)/s$.

Provided that the loop remains linear, the jitter in the VCO phase due to input noise may be computed independently of the input signal modulation. Furthermore, the mean square phase jitter will be directly proportional to input noise power under the same assumption. A simplified derivation of the magnitude of the phase jitter proceeds as follows and is valid for high signal-to-noise ratios. Let the input noise spectral density be N_o and the input carrier power be S. Then, the spectral density of the quadrature noise component is $2N_o$ (starting at zero frequency). Since the phase jitter is the quadrature noise divided by the carrier amplitude $\sqrt{2S}$, it has spectral density N_o/S, and the mean square phase jitter is found by weighting the input spectral density according to the closed loop transfer function F(s). A more detailed iterative solution has been derived and shows that the above argument is rigorously valid whenever the phase error is small, as required by the linearity postulate.[11]

The loop threshold must take into account the phase error produced by both modulation and noise. Hence, optimum loop design attempts to reach the best compromise between excessive modulation phase error which occurs with a narrow loop bandwidth and excessive noise phase error occurs with a wide loop bandwidth. The main question raised is the definition of modulation error; for example, the relative significance of peak versus rms error. In the following sections rms modulation error is emphasized; however, peak error is considered in certain special cases.

7. OPTIMUM DESIGN OF PHASE LOCK LOOP DISCRIMINATOR

A phase lock loop has been demonstrated qualitatively to function as a frequency discriminator capable of demodulating an FM signal. The quantitative performance may be computed after optimization of the loop design has been carried out. This optimization problem will be treated in two parts. The first considers the theoretical selection of the loop filter to minimize the loop threshold; the second treats the optimal selection of specific simple filters.

Actually, there are two filters of significance to loop performance. The first, or loop filter, is in the closed loop and significantly affects the threshold properties of the loop. The second, or output filter, has no effect on threshold but is required to eliminate noise outside of the base-bandwidth. It will be presumed to be an ideal filter with sharp cutoff at the highest modulating frequency and yielding an overall flat response.

The loop threshold occurs when the loop phase error becomes excessive. From a theoretical point of view, specification of allowable mean square phase error has the advantage of being mathematically tractable, and this approach will be followed below except in certain special cases where peak error is very important. The phase error arises from both signal modulation and noise, and the composite error must be taken into account when loop optimization is carried out.

Using the mean square error criterion, loop filter optimization can be carried out for any specified signal spectrum. (White noise is presumed, as usual.) This optimization is not particularly significant in practical cases where the spectrum of the signal is unknown a priori. Therefore, a reasonable approach is to design for the worst spectrum; that is, the minimax concept appropriate to the theory of games is introduced. This approach has been followed by Yovits and Jackson,[12, 13] and their work may be adapted to the phase lock loop system. The resulting optimum loop filter may be termed the game theory filter. Performance with practical filters may be compared with the game theory performance.

7.1 GAME THEORY FILTER

The game theory approach determines the filter which minimizes the mean square error for a given signal spectrum and then selects the signal spectrum which maximizes the minimum error. This is the mini-max solution or worst case. The optimization is carried out under the constraints:

(1) The filter is physically realizable.

(2) The rms frequency deviation ΔF is given as a parameter.

(3) The modulation has the maximum frequency f_m.

The last constraint specifies bandlimited modulation, the second yields the effective spectral occupancy of the RF signal. Of course, the peak frequency deviation is probably more meaningful in practice; however, an rms constraint can be treated mathematically, as previously mentioned.

The mean square phase error in the loop arises from the additive white Gaussian noise of (one sided) spectral density N_o and also because the modulation can not be tracked exactly. If the input phase modulation has a spectral density $S_i(f)$, the modulation error is

$$\sigma_s^2 = \int_o^\infty S_i(f) \left| 1 - F(j\,\omega) \right|^2 df \tag{7-1}$$

using the loop error transfer function of equation 6-6. As previously mentioned, if the phase error remains reasonably small, superposition applies, and the noise error may be computed with the closed loop transfer function of equation 6-5. Since the noise spectral density at the multiplier output has been shown to be $2N_o/(\sqrt{2S})^2 = N_o/S$, the noise error is

$$\sigma_n^2 = \frac{N_o}{S} \int_o^\infty \left|F(j\omega)\right|^2 df \tag{7-2}$$

The total mean square phase error is the sum of equations 7-1 and 7-2.

The loop threshold may be defined by the inequality

$$\sigma_s^2 + \sigma_n^2 \leqslant \sigma_e^2 \tag{7-3}$$

where σ_e^2 is appropriately chosen based on loop nonlinearity. A common choice is $\sigma_e^2 = 1$, which means an rms phase error of one radian. In general, the loop optimization consists of minimizing $\sigma_s^2 + \sigma_n^2$ subject to the constraints previously mentioned. The game theory optimization is in parts: (1) Minimize the loop phase error for a given input spectrum $S_i(f)$ and (2) Determine the "worst" spectrum; which maximizes the minimum phase error.

Present approaches to phase lock loop design specify a second-order loop transfer function and require a suitably small phase error for sine wave modulation at any frequency below the cutoff f_m. The design procedure presented in the previous paragraph is more general because arbitrary transfer functions and input spectra are allowed. However, it is also more limited, since peak modulation error is not considered.

The first part of the optimization problem has already been solved by Yovits and Jackson for the case of specified signal spectrum. Their result is

$$\left| 1 - F(j\omega) \right|^2 = \frac{(N_o/S)}{S_i(f) + (N_o/S)} \tag{7-4}$$

for the amplitude characteristic; the phase may be computed by the Bode relations,[14] since both $1-F(j\omega)$ and $F(j\omega)$ may be shown to be minimum phase. Furthermore, the minimum mean square phase error is found to be

$$(\sigma_s^2 + \sigma_n^2)_{min} = \int_o^\infty \frac{N_o}{S} \log \left[1 + \frac{S_i(f)}{N_o/S} \right] df \tag{7-5}$$

The maximum value of equation 7-5 is now to be obtained by variation of $S_i(f)$, subject to the constraints

$$S_i(f) = 0 \quad , \qquad f > f_m \tag{7-6}$$

$$\int_o^\infty f^2 S_i(f)\, df = \Delta F^2 \tag{7-7}$$

Equation 7-7 arises because the instantaneous modulation frequency is the time derivative of the input phase. There is an additional constraint related to the fact that $S_i(f)$ is a non-negative spectral density; namely,

$$S_i(f) \geqslant 0 \tag{7-8}$$

No other limitations are placed on $S_i(f)$.

Inspection of equations 7-5 and 7-7 shows that $S_i(f)$ will tend to be concentrated at the lower frequencies. For the moment, let us ignore the non-negative constraint of equation 7-8. Noting that the integrands of both equations 7-5 and 7-7 are zero for $f > f_m$, the calculus of variations may be applied to give

$$\frac{1}{(N_o/S) + S_i(f)} - \lambda f^2 = 0 \tag{7-9}$$

where λ is a Lagrangian multiplier. Solving for $S_i(f)$ and using the constraint of equation 7-7 gives

$$S_i(f) = \left(\frac{\Delta F^2}{f_m} + \frac{N_o f_m^2}{3S}\right)\frac{1}{f^2} - \frac{N_o}{S} \tag{7-10}$$

in the range $f \leqslant f_m$, of course. However, equation 7-10 does not satisfy the non-negative constraint over the entire range of deviation ratios. Requiring that $S_i(f)$ be non-negative yields the limitation.

$$\frac{S}{2N_o f_m} \geqslant \frac{1}{3(\Delta F/f_m)^2} \tag{7-11}$$

As will be seen later, this limitation is actually satisfied over the range of deviation ratios of interest. Then, substituting equation 7-10 into equation 7-5 and integrating gives

$$(\sigma_s^2 + \sigma_n^2)_{\text{max-min}} = \frac{N_o f_m}{S} \log \left[e^2\left(\frac{1}{3} + \frac{S}{N_o f_m}\,\frac{\Delta F^2}{f_m^2}\right)\right] \tag{7-12}$$

which is the desired game theory phase error, provided the inequality of equation 7-11 is true.

The game theory nature of the solution may be observed directly from the modulation error of equation 7-1, since the noise error of equation 7-2 is fixed after the transfer function is specified. Substituting equation 7-10 into equation 7-4 yields for the amplitude of the game theory filter

$$\left|1 - F(j\omega)\right|^2_{opt.} = \frac{(f/f_m)^2}{\frac{1}{3} + \frac{S}{N_o f_m} \frac{\Delta F^2}{f_m^2}} \qquad f \leqslant f_m$$

$$= \quad 1 \qquad f > f_m \tag{7-13}$$

Then, for an arbitrary input spectrum bandlimited to f_m

$$\sigma_s^2 = \int_o^{f_m} S_i(f) \left| 1 - F(j\omega)\right|^2_{opt.} df$$

$$= \frac{1}{f_m^2 \left(\frac{1}{3} + \frac{S}{N_o f_m} \frac{\Delta F^2}{f_m^2}\right)} \int_o^{f_m} f^2 S_i(f) df$$

$$= \frac{(\Delta F/f_m)^2}{\frac{1}{3} + \frac{S}{N_o f_m} (\Delta F/f_m)^2} \tag{7-14}$$

where equation 7-7 has been used. Thus, the modulation error and, therefore, the total error is independent of the spectral shape of the input modulation, as is characteristic of a game theory solution. Incidentally the difference between equations 7-12 and 7-14 yields the noise error.

The normalized threshold for a given rms deviation ratio $\Delta F/f_m$ may now be computed from equation 7-12. That is, for a given mean square error σ_e^2, one has

$$\log e^2 \cdot \left[\frac{1}{3} + \frac{S}{N_o f_m} \left(\frac{\Delta F}{f_m}\right)^2\right] = \frac{S}{N_o f_m} \quad \sigma_e^2 \tag{7-15}$$

which is a transcendental equation in the desired threshold. Above threshold, the usual FM improvement applies. While the improvement is commonly specified in terms of peak deviation ratio for a sine wave, it is $6(\Delta F/f_m)^2$ in terms of the rms deviation ratio used here. A plot of normalized threshold versus rms deviation ratio is given in figure 7-1 for the two cases $\sigma_e = 1$ and 0.5. A plot of output signal-to-noise ratio versus normalized threshold is given in figure 7-2, again for the two cases $\sigma_e = 1$ and 0.5.

As a further observation, equation (7-14) divided by the total mean square phase error gives the fraction due to modulation. This fraction is plotted in figure 7-3 and is seen to be small. It approaches zero as the deviation ratio becomes very large.

7.2 OPTIMUM SECOND ORDER LOOP

A commonly used filter for a practical phase lock loop is (to a good approximation) the ideal integrator[15]

$$H(s) = \sqrt{2K} + \frac{K}{s} \tag{7-16}$$

for which the loop transfer function is

$$F(s) = \frac{\sqrt{2K}\ s + K}{s^2 + \sqrt{2K}\ s + K} \tag{7-17}$$

The quantity $\sqrt{K}/2\pi$ is called the loop resonant frequency B_o. We wish to compare the thresholds of phase lock loops with the optimum game theory transfer function and with the second order filter of equation 7-17. This will indicate the potential improvement by going to more complicated filters

Equations 7-1 and 7-2 still apply and evaluation of the integral in equation 7-2 yields

$$\sigma_n^2 = 0.53\ \sqrt{K}\ N_o/S \quad = B_N N_o/S \tag{7-18}$$

Equation 7-18 is interpreted by defining the noise bandwidth of the loop to be

$$B_N = 0.53\sqrt{K} \tag{7-19}$$

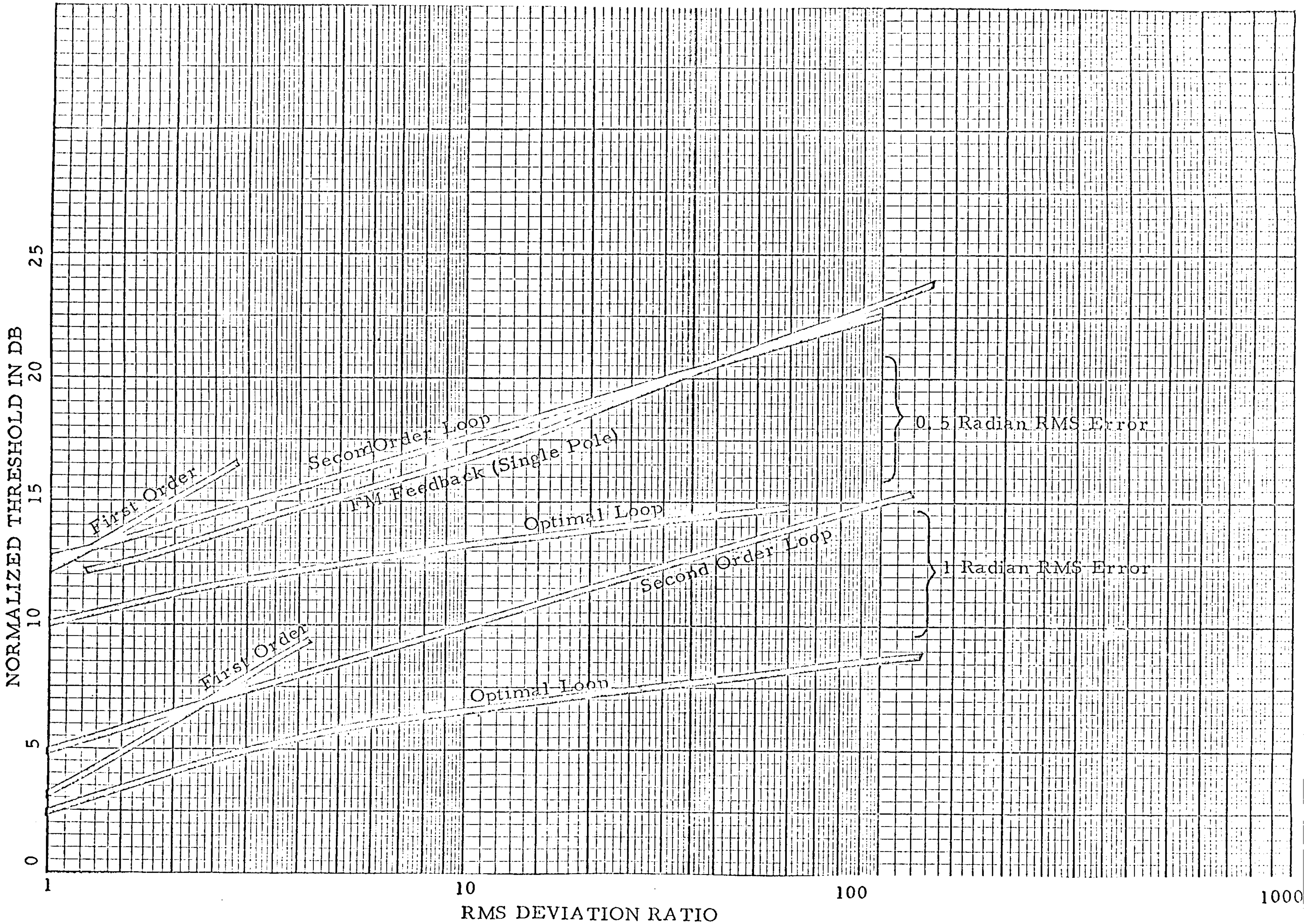

Figure 7-1. Comparison of Feedback FM Demodulators

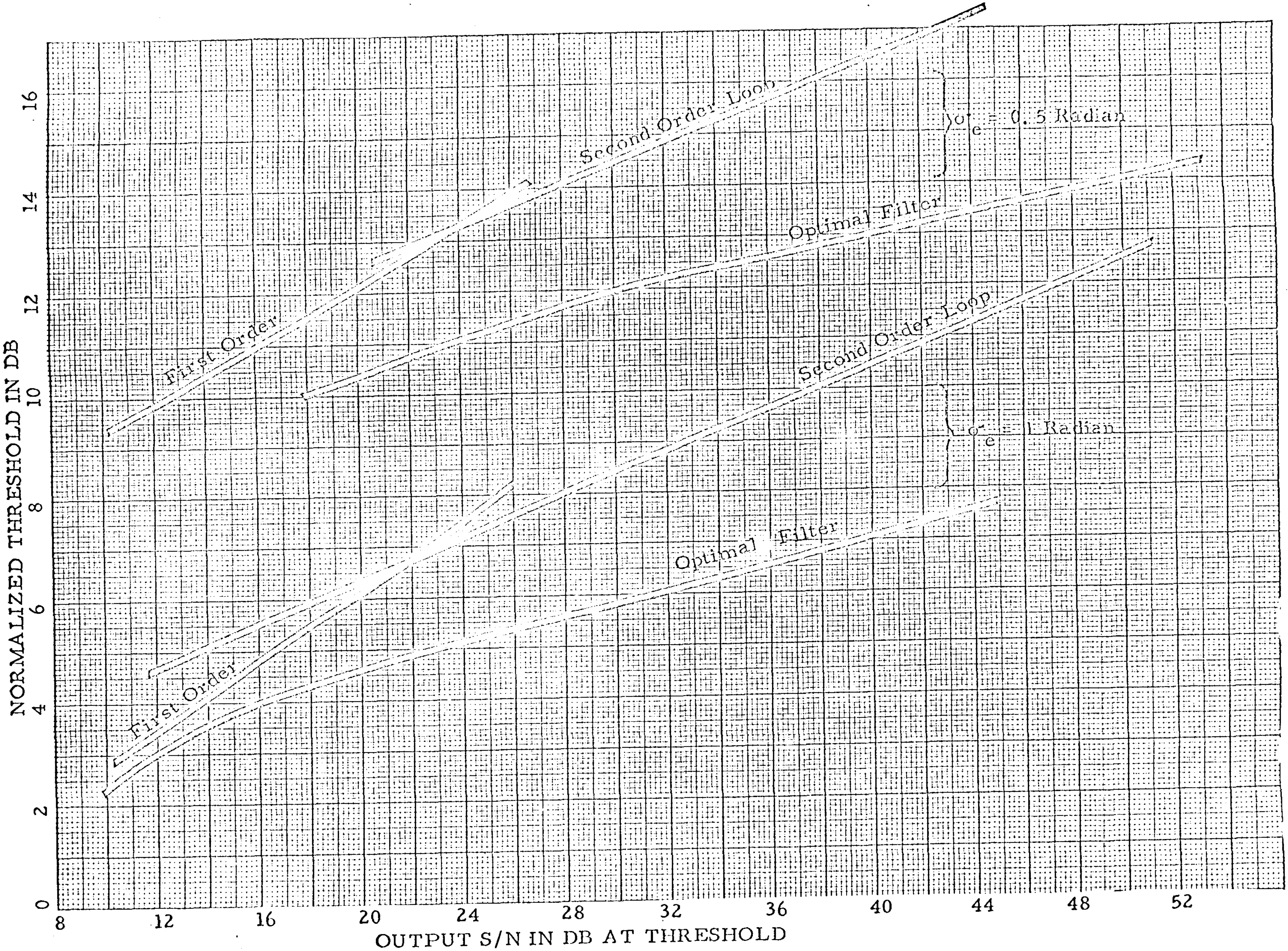

Figure 7-2. Threshold Improvement Characteristics

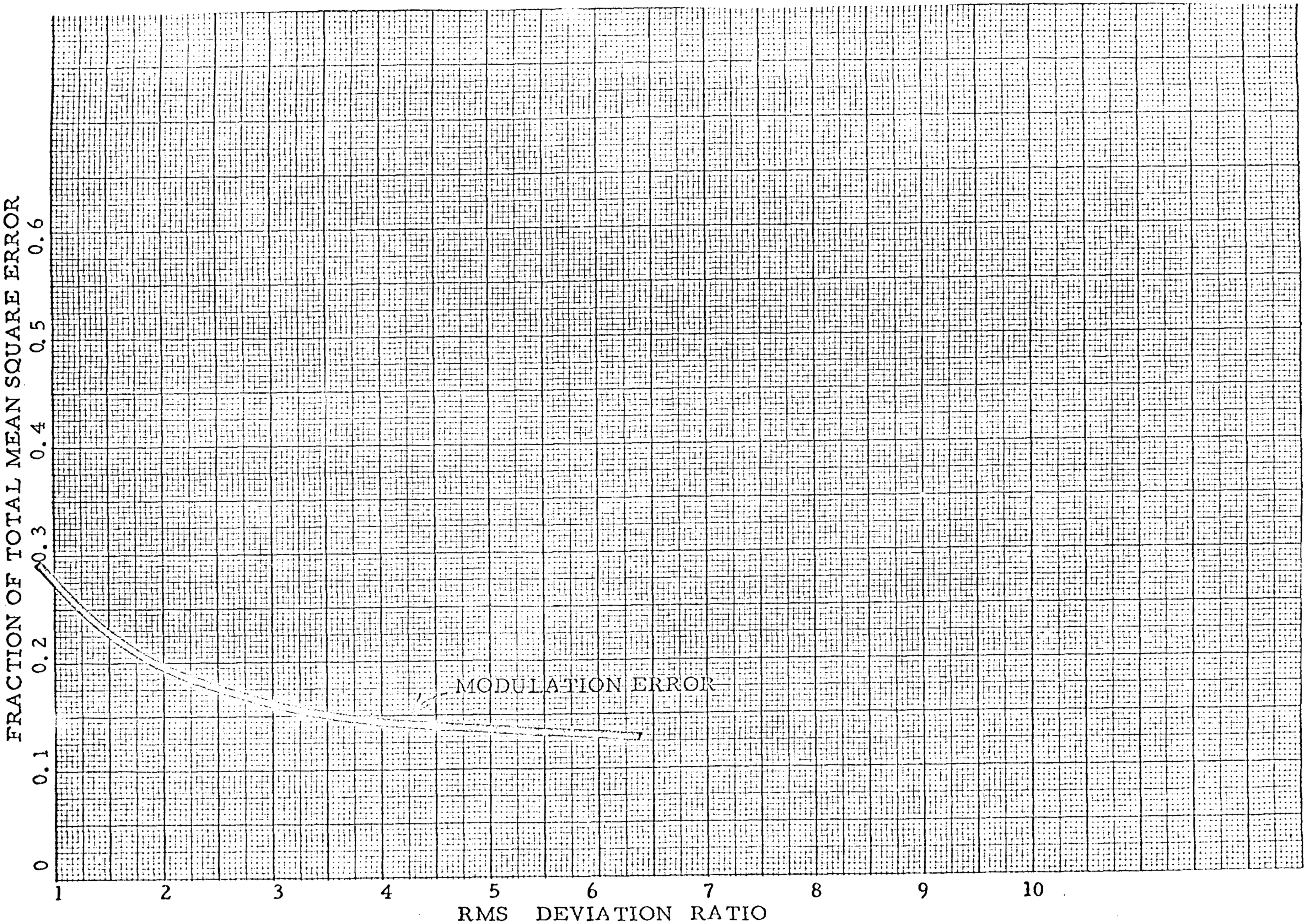

Figure 7-3. Modulation Error for $\sigma_e = 1$

For the modulation error, the "worst" case is of interest. Now, for the second-order transfer function of equation 7-17, there obtains

$$\left|1 - F(j\omega)\right|^2 = \frac{\omega^4}{K^2 + \omega^4} = \frac{f^4}{B_o^{\,4} + f^4} \tag{7-20}$$

and the maximum modulation error will occur for sinusoidal modulation at the cutoff frequency f_m, if $f_m < B_o$, or at the frequency B_o, otherwise. We assume that the former case applies. Then, the modulation error is

$$\sigma_s^{\,2} = \frac{(2\pi\,\Delta F)^2\,\omega_m^{\,2}}{K^2 + \omega_m^{\,4}} \tag{7-21}$$

For any given rms deviation ratio $\Delta F/f_m$, an optimum K can be selected to minimize $\sigma_n^{\,2} + \sigma_s^{\,2}$. Alternatively the optimization can be viewed as the minimization of S/N_o, for a specified total error. Taking the latter point of view and writing $\sigma_n^{\,2} + \sigma_s^{\,2} = \sigma_e^{\,2}$, we may solve for N_o/S, yielding

$$\frac{N_o}{S} = \frac{1}{0.53\sqrt{K}}\left[\sigma_e^{\,2} - \frac{(2\pi\Delta F)^2\,\omega_m^{\,2}}{K^2 + \omega_m^{\,4}}\right] \tag{7-22}$$

which is to be maximized by choice of K. (If $\omega_m > K$, equation 7-22 should be rewritten with the substitution K for $\omega_m^{\,2}$.) By differentiating with respect to K and equating to zero, the solution is obtained

$$K^2 = \frac{2.5}{\sigma_e^{\,2}}\,\omega_m^{\,2}\,(2\pi\,\Delta F)^2\left[1 - 0.4\,\frac{\omega_m^{\,2}\,\sigma_e^{\,2}}{(2\pi\,\Delta F)^2} + \sqrt{1 - 0.64\,\frac{\omega_m^{\,2}\,\sigma_e^{\,2}}{(2\pi\,\Delta F)^2}}\,\right] \tag{7-23}$$

and for reasonable deviation ratios, $\omega_m^{\,2} < K$, as required for validity of the solution. Substitution of equation 7-23 into equation 7-22 with $\sigma_e = 1$ and 0.5, respectively, yields the curves for the second order loop presented in figures 7-1 and 7-2.

For large deviation ratios, K becomes directly proportional to the product of cutoff frequency and deviation, so that the loop noise bandwidth increases as the square root of the deviation ratio. For this behavior, the normalized threshold is

$$\frac{S}{2N_o f_m} \doteq \pi \sqrt{\frac{\Delta F}{\sigma_e^5 f_m}} \tag{7-24}$$

and the relation between output signal-to-noise ratio and normalized threshold is

$$\left(\frac{S}{N}\right)_{out} = 0.06\, \sigma_e^{10} \left(S/2N_o f_m\right)^5_{thresh} \tag{7-25}$$

This means that a one db increase in normalized threshold, caused by an increase in deviation ratio, yields 5 db increase in output signal-to-noise ratio. Equations 6-1 and 6-4, which apply for the conventional discriminator, show that the output signal-to-noise ratio increases only by 2.2 db for each one db increase in normalized threshold. This illustrates the superiority of the phase lock loop as a discriminator.

7.3 OPTIMUM FIRST ORDER LOOP

Noting that for small modulation indices, the loop noise bandwidth is comparable to the frequency deviation, the wide tracking capability of the second order loop is not really needed. Consequently, the first order loop (H(s) = K) may be considered. Optimization on an rms error basis will be carried out here, in similarity with the approach for the game theory filter and the second order loop.

The loop error transfer function for the first order loop is

$$1 - F(s) = \frac{s}{s+K} \tag{7-26}$$

and the loop noise bandwidth is computed to be K/4. (This multiplies the spectral density N_o/S to give the mean square noise error.) For an rms frequency deviation ΔF, the modulation error is

$$\sigma_s^2 = \frac{(2\pi\Delta F)^2}{\omega^2 + K^2} \tag{7-27}$$

and the maximum modulation error occurs for $\omega = 0$.

The total mean square error, then, is

$$\frac{(2\pi\Delta F)^2}{K^2} + \frac{K}{4}\ \frac{N_o}{S} = \sigma_e^2 \tag{7-28}$$

The choice of loop gain K which maximizes N_o/S (minimum normalized threshold) is found to be

$$K = \sqrt{3}\ \ 2\pi\Delta F/\sigma_e \tag{7-29}$$

Substituting the optimum gain into equation 7-28 and solving for the normalized threshold yields the result

$$\left(\frac{S}{2N_o f_m}\right)_{thresh} = \frac{2.04}{\sigma_e^3}\left(\frac{\Delta F}{f_m}\right) \tag{7-30}$$

indicating that reducing the phase error limit σ_e by a factor of 2 raises the threshold by 9 db. Applying the usual FM gain gives

$$\left(\frac{S}{N}\right)_{out} = 1.44\ \sigma_e^6\ \left(\frac{S}{2N_o f_m}\right)^3_{thresh} \tag{7-31}$$

so that the improvement factor is better than with a second order loop only for small $(S/N)_{out}$. Equations 7-30 and 7-31 are plotted in figures 7-1 and 7-2 to facilitate the comparison of first and second order loops.

7.4 OPTIMUM MODULATION WITH GAME THEORY FILTER

Frequency modulation has been presumed primarily because of its well-known ability to trade bandwidth for signal-to-noise ratio improvement. However, using a phase lock loop demodulator does not necessarily lead to frequency modulation as the optimum theoretical choice. This optimization problem may be discussed as a generalization of the game theory solution previously presented.

The constraints on the choice of signal spectrum and method of modulation are:

1. Input modulation power is specified.
2. Required output signal-to-noise ratio is specified.

Of course, the carrier power and input noise spectral density are also fixed parameters. It is desired to minimize the mean square phase error subject to the above constraints. To simplify the computation, it will be presumed that the output phase noise from the phase lock loop is essentially flat below the maximum modulation frequency f_m.

The method of modulation may be described by a non-negative weighting function Y(f) on the loop output, which is taken to be the VCO phase, obtained by passing the output frequency through an ideal integrator. For example, if Y(f) is proportional to f^2, frequency modulation results. The inverse weighting $Y^{-1}(f)$ is employed at the modulator to give an overall flat response. Then, the constraints become

$$\int_0^{f_m} S_i(f)\, df = 1 \qquad (7\text{-}32)$$

and

$$\frac{N_o}{S} \int_0^{f_m} Y(f)\, df = \left(\frac{N}{S}\right)_{out} \qquad (7\text{-}33)$$

where $S_i(f)$ must be non-negative. Now, the game theory solution consists of assuming the worst signal spectrum, which maximizes the phase error,

$$\sigma_e^2 = \int_0^{f_m} \frac{N_o}{S} \log \left[1 + \frac{S}{N_o} \frac{S_i(f)}{Y(f)}\right] df \qquad (7\text{-}34)$$

is used with the best weighting, which minimizes the phase error. (Note that S_i/Y gives the input phase modulation spectrum.)

Applying the calculus of variations yields the pair of equations

$$\frac{1/Y(f)}{(N_o/S) + S_i(f)/Y(f)} - \lambda_1 = 0 \qquad (7\text{-}35)$$

and

$$\frac{-S_i(f)/Y^2(f)}{(N_o/S) + S_i(f)/Y(f)} - \lambda_2 \frac{N_o}{S} = 0 \qquad (7\text{-}36)$$

The solution of equations 7-35 and 7-36 is expected to have the desired saddle point behavior and turns out to be simply

$$\begin{aligned} Y(f) &= \text{constant} \\ S_i(f) &= \text{constant} \end{aligned} \qquad (7\text{-}37)$$

over the modulation bandwidth, and the non-negative requirement is satisfied. The interpretation of the solution is that pure phase modulation is the optimum method of modulation to be used with a phase lock loop incorporating a game theory filter.

Evaluating the constants from the constraints, equations 7-32 and 7-33, yields for the game theory phase error

$$\sigma_e^2 = \frac{f_m N_o}{S} \log \left[1 + \left(\frac{S}{N}\right)_{out} \right] \qquad (7\text{-}38)$$

or solving for $(S/N)_{out}$

$$\left(\frac{S}{N}\right)_{out} = e^{\sigma_e^2 S/N_o f_m} - 1 \qquad (7\text{-}39)$$

For comparison, equation 7-15, which is the game theory solution for frequency modulation, may be written as

$$\left(\frac{S}{N}\right)_{out} = 0.405\ e^{\sigma_e^2 S/N_o f_m} - 1 \qquad (7\text{-}40)$$

Thus, for a high deviation, FM yields 4 db less $(S/N)_{out}$ than PM at equal normalized thresholds, when the optimum game theory filters are employed for each.

It is interesting to note that the second order loop actually is inferior for PM, and FM is, therefore, a more suitable modulation choice for the practical case. The reason is that the modulation error is maximum at the highest frequency, at which FM causes a smaller phase deviation.

The optimum game theory filter for phase modulation is found from equation 7-4 to be

$$\left|1 - F(j\omega)\right| = \frac{1}{\sqrt{1 + (\frac{S}{N})_{out}}} \qquad f < f_m \qquad (7\text{-}41)$$

$$= 1 \qquad f > f_m$$

hence is an attenuation step. The phase corresponding to this amplitude function is

$$\beta = \frac{\log \sqrt{1 + (S/N)_{out}}}{\pi} \log \left|\frac{f + f_m}{f - f_m}\right| \qquad (7\text{-}42)$$

which displays a logarithmic infinity at $f = f_m$.

From a practical point of view, the "optimum" filter of equation 7-41 is actually unusable because it does not approach zero as $\omega \to 0$. This means that a frequency error due to mistuning of the VCO will produce an infinite phase error. That is, the optimum filter for phase modulation presumes perfect knowledge of the frequency, an impossibility in practice. This difficulty can be eliminated with a negligible theoretical penalty by making $1 - F(j\omega)$ approach zero at very low frequencies. However, the transient response of the loop would then, very likely, be unacceptable.

7.5 OPTIMUM MODULATION FOR SECOND ORDER LOOP

We may perform a restricted version of the analysis in section 7.4 by directing attention at the second order loop. The problem is now to determine the optimum weighting function $Y(f)$ for a fixed loop filter, again assuming the constraints of equation 7-32 and 7-33. The loop noise bandwidth is held fixed, and the maximum modulating frequency is assumed much less than the loop resonant frequency. Then, the noise error is fixed, and we wish to minimize the modulation error for the worst input spectrum.

Mathematically, the function $Y(f)$ is to be chosen so that the maximum value of

$$\int_{o}^{f_m} \frac{f^4}{B_o^4} \frac{S_i(f)}{Y(f)} \, d f \qquad (7\text{-}43)$$

for arbitrary $S_i(f)$ is minimized. From the constraint of equation 7-32 and concentrating $S_i(f)$ at the worst frequency, we obtain the requirement

$$\underset{\text{over all } Y(f)}{\text{Minimize}} \quad \left. \frac{f^4}{B_o^4 \, Y(f)} \right]_{\substack{\text{Max} \\ 0<f<f_m}} \qquad (7\text{-}44)$$

subject to the constraint of equation 7-33. The result is that

$$Y(f) = \lambda f^4 \qquad (7\text{-}45)$$

This means that the modulation should be doubly integrated before phase modulating the carrier. Since frequency modulation corresponds to only a single integration, it is not optimum for the worst case modulation spectrum.

From a practical point of view, doubly integrating the modulation corresponds to pre-emphasis of the low frequencies in an FM system. The result is that the peak frequency deviation for sinewave modulation approaches infinity as the modulating frequency approaches zero. This system behavior is not acceptable in practice. Hence, the single integration of frequency modulation may be considered as a practical optimum modulation for second order loops.

It is interesting to note that frequency modulation is theoretically optimum when the input modulation spectrum is flat. In this case, we wish to minimize the modulation error, given by

$$\int_{o}^{f_m} \frac{f^4}{B_o^4} \frac{df}{Y(f)} \qquad (7\text{-}46)$$

by choice of Y(f) subject to the constraint of equation 7-33. The calculus of variations gives the result

$$Y(f) = \lambda f^2 \tag{7-47}$$

which corresponds to pure frequency modulation, of course.

8. APPROXIMATE THRESHOLD OF FM FEEDBACK DEMODULATOR

The FM feedback demodulator has been advanced as capable of operation at a lower threshold than a phase lock loop discriminator because it can tolerate a larger modulation error.[16] This statement is subjected to scrutiny in the following approximate analysis of an FM feedback demodulator.

Enloe has stated the desirability of incorporating a "baseband" filter in the feedback path and making the IF bandwidth as wide as possible consistent with an adequate signal-to-noise ratio at the frequency detector. Consequently, we may represent the FM feedback demodulator as shown in figure 8-1. The dotted IF filter indicates that it is so wide that it does not need to be included in the transfer function. Noise is not discussed, for the moment.

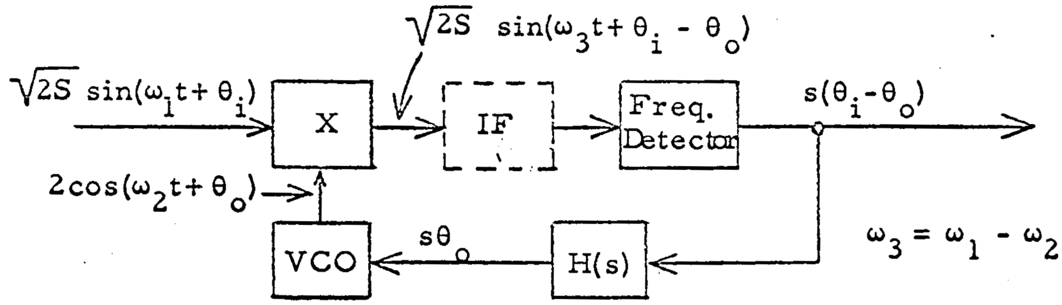

Figure 8-1. FM Feedback Discriminator

The equation describing loop behavior in accordance with the above assumptions is

$$s\theta_o = H(s)s(\theta_i - \theta_o) \tag{8-1}$$

so that the transfer function for instantaneous frequency is

$$\frac{s\theta_o}{s\theta_i} = \frac{H(s)}{1 + H(s)} \tag{8-2}$$

The instantaneous frequency at the mixer output is given by

$$\frac{s(\theta_i - \theta_o)}{s\theta_i} = \frac{1}{1 + H(s)} \tag{8-3}$$

These equations may now be employed for sine wave frequency modulation.

If the input carrier is frequency modulated at the frequency f, the input phase is

$$\theta_i = \frac{\Delta F}{f} \sin \omega t \tag{8-4}$$

where ΔF is the peak frequency deviation. Then, the peak frequency deviation at the mixer output is

$$\frac{1}{2\pi} \left| s(\theta_i - \theta_o) \right|_{peak} = \frac{\Delta F}{\left| 1 + H(j\omega) \right|} \tag{8-5}$$

This is needed ultimately to specify the required IF bandwidth.

The simplest filter in the feedback path has a single pole and is

$$H(s) = \frac{K}{1 + \tau s} \tag{8-6}$$

which contains a gain factor K and has a 3-db cutoff frequency of $1/2\pi\tau$ in cps. In terms of this filter, the frequency transfer function is

$$\frac{s\theta_o}{s\theta_i} = \frac{K}{K + 1} \quad \frac{1}{1 + \tau s/(K + 1)} \tag{8-7}$$

showing that the closed loop transfer function has a wider bandwidth than the feedback filter. The noise bandwidth is that of a single pole filter and is

$$B_N = \int_0^\infty \frac{df}{1 + (\tau\omega)^2/(K + 1)^2} = \frac{K + 1}{4\tau} \text{ cps} \tag{8-8}$$

referred to the DC gain, which is $K/(K + 1)$.

The compressed frequency deviation at the mixer output is also of interest and is found from equation 8-5 to be

$$\frac{1}{2\pi}\left|s(\theta_i-\theta_o)\right|_{peak} = \frac{\Delta F}{K+1}\sqrt{\frac{1+(\omega\tau)^2}{1+(\omega\tau)^2/(K+1)^2}} \qquad (8\text{-}9)$$

Equation 8-9 demonstrates that the frequency deviation in the IF is, in fact, compressed by the factor K + 1 for low modulating frequencies. However, the deviation in the IF starts increasing nominally at $\omega\tau = 1$, a result of frequency limitation in the feedback path. Thus, we may write, optomistically,

$$f_m = \frac{1}{2\pi\tau} \qquad (8\text{-}10)$$

as the maximum allowable modulating frequency. The deviation ratio of the input carrier is, therefore,

$$\frac{\Delta F}{f_m} = 2\pi\Delta F\tau \qquad (8\text{-}11)$$

Now, the effects of noise must be introduced.

The input noise is presumed to be white Gaussian with the (one-sided) spectral density N_o in watts/cps. The usual assumption is that the quadrature component of the noise produces phase jitter on the input carrier. Because the quadrature noise has spectral density $2N_o$, and the phase jitter is this noise divided by $\sqrt{2S}$, the jitter has spectral density $2N_o/(\sqrt{2S})^2 = N_o/S$. Hence, the mean square phase jitter due to noise in the VCO output is

$$\overline{(\theta_i - \theta_o)^2}_{noise} = \frac{N_o}{S}\left(\frac{K}{K+1}\right)^2 B_N = \frac{1}{10} \qquad (8\text{-}12)$$

The last equality of equation 8-12 uses the empirical result that the "closed-loop" threshold in the FM feedback demodulator occurs when the rms phase jitter is about 1/3 radian.(16)

It is now necessary to introduce the IF filter, under the previously mentioned assumption that its width is so great that eqs. 8-7 and 8-8 are not affected. The approximate criterion may be used that the 3-db bandwidth of the filter equals the (compressed) peak-to-peak frequency deviation plus twice the maximum modulating frequency, or $(1/\pi\tau) + 2\Delta F/(K+1)$. The noise bandwidth of a single pole IF is $\pi/2$ greater, so that the signal-to-noise ratio at the frequency detector input is

$$\left(\frac{S}{N}\right)_{\text{Freq det.}} = \frac{S}{N_o\left[\frac{1}{2\tau} + \frac{\pi\Delta F}{K+1}\right]} \tag{8-13}$$

and the deviation ratio in the IF is, of course, $2\pi\Delta F\tau/(K+1)$. The threshold for an ordinary frequency detector may be used here to establish the minimum allowable value of eq 8-13 and, therefore, the maximum ΔF. This threshold curve is presented in figure 6-1. Using the approximation of equation 6-2, the frequency detector or "open-loop" threshold may be expressed as

$$\left(\frac{S}{N}\right)_{\text{Freq. det. thresh}} = 3.8\left[\frac{\pi}{2}\left(\frac{2\pi\Delta F\tau}{K+1} + 1\right)\right]^{0.72} \tag{8-14}$$

The proper demodulator design is achieved by equating the feedback threshold given by eq 8-12 with the frequency detector threshold of equation 8-14 and substituting eq 8-8 for the loop noise bandwidth. The result is

$$\left(\frac{2\pi\Delta F\tau}{K+1} + 1\right)^{1.72} = 0.955\,\frac{K^2}{K+1} \tag{8-15}$$

which may be solved for the deviation ratio as a function of K, as follows.

$$2\pi\Delta F\tau = \frac{\Delta F}{f_m} = 0.973\,K^{1.16}(K+1)^{0.42} - (K+1) \tag{8-16}$$

Now, eqs 8-8 and 8-10 yield the relation

$$\frac{B_N}{f_m} = \frac{\pi}{2}(K+1) \tag{8-17}$$

which combined with eq. 8-12 gives

$$\left(\frac{S}{2N_o f_m}\right)_{\text{Thresh}} = 2.5\pi\,K^2/(K+1) \tag{8-18}$$

Combining equations 8-16 and 8-18 yields the normalized threshold as a function of the deviation ratio. This function is plotted in figure 7-1. It is seen that the FM feedback demodulator is inferior to the second order loop, provided that an rms phase error greater than 0.5 rad can be tolerated in the phase lock loop. This is true in practice.

If the output signal-to-noise ratio at threshold is computed using the usual gain, it is found that for each one db increase in normalized threshold, the output signal-to-noise ratio increases by 4.2 db, for wide deviation ratios. Again, this is inferior to the second order phase lock loop.

It may be observed that the FM feedback discriminator performance should be improved by substituting a better discriminator in place of that shown in figure 8-1. In fact, a phase lock loop can directly replace the IF filter and discriminator, thereby lowering the "open-loop" threshold. In that case, the FM feedback discriminator actually reduces to a phase lock loop; however, the loop filter is usually more complex than normally employed.

To illustrate this, suppose that the phase lock loop filter is $H_1(s)$, as shown in figure 8-2.

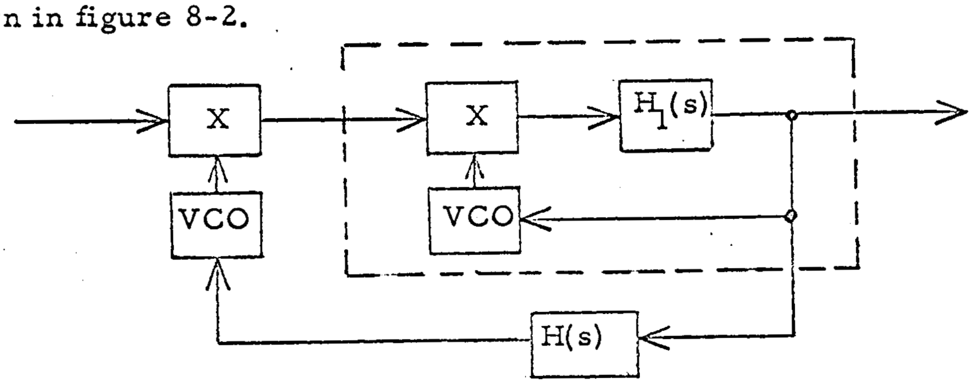

Figure 8-2. FM Feedback with Phase Lock Loop Discriminator

Because the feedback paths are in parallel, the VCO's may be combined, and the effective phase lock loop filter is

$$H_e(s) = H_1(s)\,(1 + H(s)) \tag{8-19}$$

For example, suppose that the phase lock loop discriminator is first order, so that $H_1(s) = K_1$. Then, if the baseband filter is single pole, as specified by equation 8-6, the effective loop filter is

$$H_e(s) = K_1\left(1 + \frac{K}{1+\tau s}\right) = K_1(1+K)\,\frac{1+s\tau/(1+K)}{1+s\tau} \qquad (8\text{-}20)$$

which is the conventional second-order filter with a finite integrator. If, on the other hand, the inside loop is second order, the resulting loop would effectively be third order.

Thus, the conclusion is reached that an FM feedback discriminator with any given filter is certainly inferior to a phase lock loop with an appropriate higher order filter. The higher order filter is easily realized by the double loop arrangement of a second-order loop and baseband feedback.

It may also be concluded, because the amount of tolerable phase noise appears to be lower for the FM feedback loop with a conventional discriminator, that even the second-order phase lock loop will yield a lower normalized threshold than an FM feedback demodulator with a reasonably simple baseband filter.

9. APPLICATION TO TELEVISION SIGNAL TRANSMISSION

The game theory optimization discussed in section 7 to minimize the maximum mean square phase error due to both signal modulation and noise is directly applicable when the signal modulation is stochastic and average power limited and a mean square phase error criterion is truly significant. It will be noted that a phase error minimization is significant because of the thresholding characteristics of a phase lock loop; namely, excessive phase error (exceeding about $\pi/2$ radians) will cause loss of phase lock.

With a practical application of FM techniques to television transmission the constraints are somewhat different than mean square, as described above. Although the modulation is stochastic, its peak-to-peak excursion is the specified quantity since this determines the spectrum occupied by the FM carrier. Similarly, the maintenance of phase lock in the demodulator is dependent more on avoiding a large instantaneous peak in the phase error than on insuring a low rms value. Thus, a further investigation of optimum loop design is warrented, in the light of practical television signal transmission.

One may ask, first, the relation between peak-to-peak frequency deviation of the carrier and the rms deviation employed in the previous analysis. Clearly, this depends on the amplitude statistics of the modulation; for example, with a uniform distribution, the peak-to-peak deviation is $2\sqrt{3}$ times the rms deviation. However, with sine wave modulation, the factor is $2\sqrt{2}$, and the limit is a factor of 2 for a square wave. (The latter can still be bandlimited if its frequency is low compared with the cut-off.) Thus, for the latter, the mean square modulation error will be 3 times greater than for a uniformly distributed signal, when the peak-to-peak deviation is the same for both.

9.1 PEAK ERROR IN SECOND ORDER LOOP

The second order loop will be discussed first. The peak phase error produced by a change in the signal modulation is of interest and may easily be computed from the linearized error transfer function

$$\frac{\theta_i - \theta_o}{\theta_i} = \frac{s^2}{s^2 + \sqrt{2K}\; s + K} \tag{9-1}$$

For a frequency step δf, the peak phase error is[17]

$$(\theta_i - \theta_o)_{peak} = 2.86 \frac{\delta f}{\sqrt{K}} \tag{9-2}$$

Inspection of equation 9-2 shows that an excessive phase error will occur for a wide deviation frequency step, unless the loop bandwidth is greater than would be needed for bandlimited modulation. Thus, an instantaneous frequency step should be prevented by appropriate filtering.

The effect of bandlimiting to a cutoff may be represented by a finite rise time, so that a modulation change is characterized by a frequency ramp.[18] The peak phase error is found to be only slightly greater (6 percent) than the steady state phase error, which may be obtained directly from equation 9-1 as

$$(\theta_i - \theta_o)_{peak} \cong \frac{2\pi \dot{f}}{K} \tag{9-3}$$

where $\dot{f}$ is the rate of change of frequency in cps/sec. Then, if the peak-to-peak frequency deviation is ΔF_{p-p} and occurs in the Nyquist interval $1/2\, f_m$, the phase error becomes

$$(\theta_i - \theta_o)_{peak} \cong \frac{4\pi f_m}{K} \Delta F_{p-p} \tag{9-4}$$

From equation 7-20, the peak error for sinusoidal modulation at the frequency f_m may be computed; the result is

$$(\theta_i - \theta_o)_{peak} = \frac{(2\pi f_m)^2}{\sqrt{K^2 + (2\pi f_m)^4}} \; \frac{\Delta F_{p-p}}{2 f_m} \tag{9-5}$$

where the peak-to-peak frequency deviation again is ΔF_{p-p}. The ratio of equations 9-5 and 9-4 is

$$\frac{(\theta_i - \theta_o)_{sine}}{(\theta_i - \theta_o)_{ramp}} = \frac{\pi}{2} \; \frac{1}{\sqrt{1 + (2\pi f_m)^4/K^2}} \cong \frac{\pi}{2} \qquad (9\text{-}6)$$

for reasonable deviation ratios. Equation 9-6 shows that sinewave modulation at the cutoff frequency imposes a more severe burden on the phase lock loop than a ramp with a rise time equal to the Nyquist interval ($1/2f_m$).

An alternative and practical design procedure for the second order loop may directly be carried out in terms of peak transient response to a frequency ramp at the maximum slope. The peak phase error is obtained from equation 9-4 in terms of rms duration ΔF, as employed in section 7, by the substitution

$$\Delta F_{p-p} = 2\sqrt{2}\ \Delta F \qquad (9\text{-}7)$$

The design criterion requires a reasonably low probability that noise will cause the phase error to become excessively large, the usual limit being taken as $\pi/2$. That is, we require

$$\frac{8\pi\sqrt{2}\ f_m \Delta F}{K} + \gamma\sqrt{\frac{N_o}{S}\ 0.53\sqrt{K}} = \frac{\pi}{2} \qquad (9\text{-}8)$$

where γ is a factor depending on the desired low probability of a large phase error. A value of about 2 appears about right.

The loop constant K may be chosen to optimize the normalized threshold by making $S/2N_o f_m$ as low as possible. Solving for the mormalized threshold and differentiating with respect to K yields the optimum result

$$K = 2.9\ \omega_m (2\pi\Delta F) \qquad (9\text{-}9)$$

Equation 9-9 may be compared with equation 7-23, which may be written

$$K \cong \frac{2.24}{\sigma_e}\ \omega_m\ (2\pi\Delta F) \qquad (9\text{-}10)$$

for deviation ratios not too small. The comparison shows that the two solutions, although based on different criteria, are essentially identical, except for a numerical constant. The comparison further suggests that the loop threshold is probably defined by $\sigma_o = 0.8$, which is in the expected range between 0.5 and 1.0 radian.

9.2 PEAK ERROR IN THIRD ORDER LOOP

The analysis in section 7 suggests that improved performance will be realized with a phase lock loop incorporating higher order filters. The third-order loop which is derived on the basis of loop response to a frequency ramp(15) yields a transfer function of the form

$$F(s) = \frac{2Bs^2 + 2B^2 s + B^3}{s^3 + 2Bs^2 + 2B^2 s + B^3} \qquad (9\text{-}11)$$

and the magnitude of the error transfer function is

$$\left| 1 - F(j\omega) \right|^2 = \frac{\omega^6}{B^3 + \omega^6} \qquad (9\text{-}12)$$

Calculation of the loop noise bandwidth gives

$$B_N = 0.833\ B \qquad (9\text{-}13)$$

As in section 9.1, a frequency ramp is presumed for the input modulation, and the peak transient phase error is desired. With a slope $\dot{f}$ and quiescent conditions initially, the transient is computed to be

$$\theta_i(t) - \theta_o(t) = \frac{2\pi\dot{f}}{B^2} \left\{ e^{-Bt} - 1.152\ e^{-Bt/2} \cos(.866\ Bt + \frac{\pi}{6}) \right\} \qquad (9\text{-}14)$$

which has a peak value of

$$(\theta_i - \theta_o)_{peak} = 0.4 \frac{2\pi\dot{f}}{B^2} = 0.277\ \frac{2\pi\dot{f}}{B_N^2} \qquad (9\text{-}15)$$

Noting that the noise bandwidth of a second order loop is $0.53\sqrt{K}$, equation 9-3 gives

$$(\theta_i - \theta_o)_{peak} = 0.28 \frac{2\pi \dot{f}}{B_N^2} \quad \text{(For second order loop)} \tag{9-16}$$

Comparison of equations 9-15 and 9-16 shows that for equal noise bandwidths, the third order loop can not track a significantly greater frequency slope than the second order loop, hence does not offer a significant performance advantage. However, because the steady-state phase error is zero the mean square error can be made lower by using a third order loop.

Of course, there is an additional practical disadvantage of a third-order filter; namely, that if the loop gain is reduced sufficiently, the loop becomes unstable.

9.3 PEAK ERROR IN FIRST ORDER LOOP

In section 7-3, the first order loop is found to have a slightly lower threshold than the second order loop for low deviation ratios, according to the mean square optimization approach. As with the second order loop, we may consider consider an alternative optimization based on transient response to a frequency ramp input. However, the ramp must be confined within the peak frequency deviation limits, since the first order loop can not track over a wide range, and the steady-state phase error with mistuning.

The single pole nature of the phase error transfer function means no overshoot; hence, the peak phase error is the steady-state error at the peak deviation, or

$$(\theta_i - \theta_o)_{peak} = \frac{\sqrt{2}\,(2\pi\,\Delta F)}{K} \tag{9-17}$$

Since the noise bandwidth of the first-order loop is K/4, we may write the threshold condition as

$$\frac{\sqrt{2}\,(2\pi\Delta F)}{K} + \gamma\sqrt{\frac{N_o}{S}\,\frac{K}{4}} = \pi/2 \tag{9-18}$$

The threshold is minimized by the solution

$$K = 2.7\,(2\pi\Delta F) \tag{9-19}$$

Comparison of equations 9-19 and 7-29 shows that there is only a numerical factor separating the two solutions, and further suggests $\sigma_e = 0.65$ radian as an approximate threshold specification. Thus, the first order loop is penalized somewhat more by transient phase error than the second order loop, offsetting the apparent advantage for low deviation ratios, to some extent.

9.4 MODIFIED SECOND ORDER FILTER

The second order filter with a perfect integrator has the particular advantage for many frequency tracking applications that the steady state phase error is zero despite a frequency error due to Doppler or mistuning. By using an imperfect integrator, an improved filter design may be realized, whereby the loop threshold is reached at a lower signal-to-noise ratio, at least according to a mean square optimization criterion.

Maintaining the damping factor of 0.707, the most general second order loop error transfer function is

$$1 - F(s) = \frac{s(s + \beta\sqrt{2K})}{s^2 + \sqrt{2K}\, s + K} \tag{9-20}$$

This function causes F(s) to become small as $s \to \infty$ and, also, gives a finite phase error with a steady frequency offset. The loop noise bandwidth is

$$B_N = \int_0^\infty \left| F(j\omega) \right|^2 df = \frac{1 + 2(1-\beta)^2}{4\sqrt{2}} \sqrt{K} \tag{9-21}$$

hence, increasing β from zero to unity reduces the loop noise bandwidth by a factor of 3. On the other hand, with sinewave modulation at the rms frequency deviation ΔF, the mean square modulation error is

$$\sigma_s^2 = \frac{(2\pi\Delta F)^2(\omega_m + \beta^2 2K)}{\omega_m^4 + K^2} \tag{9-22}$$

which increases with β.

An optimization of the filter is carried out by adjusting β and K to minimize the normalized threshold. A general solution has not been obtained; however, a numerical optimization was carried out for the particular case $\Delta F/f_m = 2$, assuming a one radian rms error. In this case, $\beta = 0.5$ and $\sqrt{K}/\omega_m = 2.9$ is the approximate optimum design, and the loop threshold is decreased by 1.1 db compared with $\beta = 0$. The noise bandwidth corresponding to $\beta = 0.5$ is $4.81 f_m$. It is $6.85 f_m$ for $\beta = 0$.

The loop filter H(s) corresponding to equation 9-20 is found to be

$$H(s) = \frac{\sqrt{2K}\,(1-\beta)\,s + K}{s + \beta\sqrt{2K}} \tag{9-23}$$

showing that the modified filter corresponds to use of a finite, rather than infinite, time-constant integrator. This means, of course, a maximum allowable mistuning. Equation 9-20 shows that the steady state phase error at the peak deviation $\sqrt{2}\,\Delta F$ is

$$(\theta_i - \theta_o)_{peak} = 2\pi\Delta F(2\beta/\sqrt{K}) \tag{9-24}$$

For the case $\Delta F/f_m = 2.0$ and $\beta = 0.5$, the steady state phase error is computed to be 0.69 radian at the peak deviation. This is well within the design range of one radian.

For an alternate optimization, the transient response to a ramp modulation may be determined. In contrast to the case $\beta = 0$, the steady-state phase error for a frequency ramp is infinite, so that the deviation bounds must be included. Assuming a peak-to-peak frequency swing of $2\sqrt{2}\,\Delta F$ in the Nyquist interval $1/2f_m$, the ramp slope is $0.9\,(2\pi\Delta F)(2\pi f_m)$. Again, considering the case $\Delta F/f_m = 2$ and $\beta = 0.5$, the transient error is presented in figure 9-1. It is seen that the peak error is only slightly greater than the steady-state error; just as is true for the response of the ordinary second order filter to a frequency ramp.

The optimization will be carried out for $\Delta F/f_m = 2$ and $\beta = 0.5$, assuming (optimistically) that the peak phase error equals the steady state error. Then, we may write

$$2.0\,\frac{\omega_m}{\sqrt{K}} + \gamma\sqrt{\frac{N_o}{S}\,0.265\,\sqrt{K}} = \frac{\pi}{2} \tag{9-25}$$

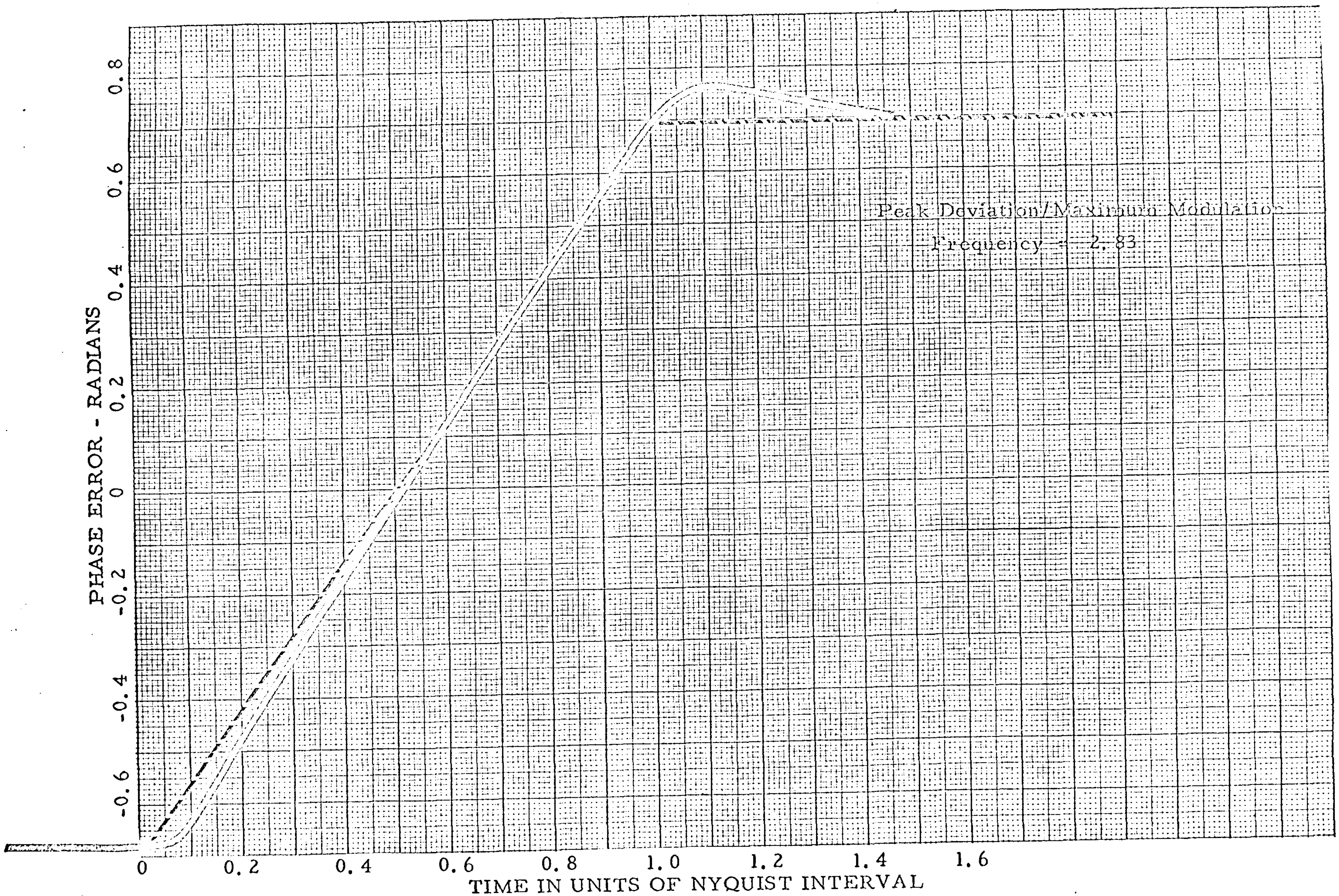

Figure 9-1. Transient Error for Modified Loop

The value of K which minimizes the threshold is 14.7 ω_m^2, and the resulting normalized threshold is

$$\frac{S}{2N_o f_m} = 2.88\,\gamma^2 \qquad (9\text{-}26)$$

However, substitution of equation 9-9 into 9-8 yields the value 2.53 γ^2 for the normalized threshold of the unmodified second order loop, indicating that the modified filter, although apparently lowering the threshold on an rms error basis, actually raises the threshold computed on the basis of transient error. Thus, it is concluded that the ordinary second order filter is to be preferred.

9.5 PEAK ERROR WITH PHASE MODULATION

The fact that a phase lock loop tracks the phase of the incoming carrier theoretically allows the use of wide deviation phase modulation. The loop output is then integrated to yield the VCO phase. It will be observed that any mistuning of the VCO means a DC offset to maintain phase lock, so that a ramp results after integration. However, for small mistuning, the ramp can be corrected by DC restoration, and this difficulty is not considered further. Note that phase modulation has already been shown in section 7.5 to be inferior to frequency modulation for the second order loop.

Mean square modulation error is discussed first. For the second order loop with rms phase deviation $\Delta\theta$, the maximum mean square error occurs at the highest modulation frequency and is

$$\sigma_s^2 = \frac{\omega_m^4}{\omega_m^4 + K^2}\,\Delta\theta^2 \qquad (9\text{-}27)$$

which is identical with the error for FM having a deviation ratio $\Delta F/f_m = \Delta\theta$.

The output signal is a sine wave of rms amplitude $\Delta\theta$. The output noise has spectral density N_o/S, since it is identical with the VCO phase jitter. Hence,

$$\left(\frac{S}{N}\right)_{out} = \Delta\theta^2\,\frac{S}{N_o f_m} \qquad (9\text{-}28)$$

However, at the same threshold, frequency modulation yields 4.8 db additional output signal-to-noise ratio. It is easy to see from equation 9-27 that this comparison applies with any loop filter for which the modulation error is greatest at ω_m for both PM and FM.

An even greater disadvantage accrues to PM when the transient response is taken into account. The signal modulation is now presumed to be a phase ramp which covers the range $2\sqrt{2}\ \Delta\theta$ in the Nyquist interval $1/2f_m$. Then from equation 9-2, the peak phase error is found to be

$$(\theta_i - \theta_o)_{peak} = \frac{2.86}{B_N/0.53}\left(\frac{2\sqrt{2}}{\pi}\Delta\theta f_m\right)$$

$$= 1.37\,\Delta\theta\, f_m/B_N \tag{9-29}$$

To insure a reasonably low probability of exceeding the allowable phase error, the rms phase jitter should be reasonably small, in accordance with

$$\frac{1.37\,\Delta\theta\, f_m}{B_N} + \gamma\sqrt{B_N N_o/S} = \pi/2 \tag{9-30}$$

where γ depends on the desired probability of exceeding $\pi/2$.

The loop noise bandwidth may be selected to maximize N_o/S, with the result

$$B_N = 2.6\,\Delta\theta f_m \tag{9-31}$$

and

$$\left(\frac{S}{2N_o f_m}\right)_{Thresh} = 1.18\gamma^2\,\Delta\theta \tag{9-32}$$

substituting equation (9-32) into equation (9-28) gives

$$\left(\frac{S}{N}\right)_{out} = \frac{1.44}{\gamma^4}\left(\frac{S}{2N_o f_m}\right)^3_{Thresh} \tag{9-33}$$

Equation 9-33 shows that $(S/N)_{out}$ improves as only the third power of the normalized threshold. Hence, as with the first order loop for FM, phase modulation will be preferred with the second order loop only for low $(S/N)_{out}$. This is not in violation with previous conclusions, since they assumed reasonably high deviations. Furthermore, equation 9-33 shows that the loop is very inferior for phase modulation at high deviation; in fact, more so than as predicted by rms error criteria.

9.6 PHASE LOCK LOOP DESIGN CURVES FOR TELEVISION TRANSMISSION

The general conclusion of the analyses preceding in section 9 is that the second order loop appears to be the optimum practical filter. Design curves have been presented slanted for a different purpose. To convert them to a more practical form, peak-to-peak frequency deviation may be employed to define RF spectrum occupancy, and the ratio of peak video signal to rms noise should be used to define picture quality. Since the worst signal for the second order loop is sine wave modulation, conversion to the practical parameters is immediate. For a standard video signal, minimum amplitude is 12.5 percent of peak amplitude of the synch pulses. However, we do not have to adhere to this standard, since sync pulses may be distinguished by other means than amplitude, as will be described later. Then, the peak-to-peak power is 9 db above the average power for a sine wave, and the peak-to-peak frequency deviation is 2.83 times the rms deviation. Figures 7-1 and 7-2 are replotted on this basis for the first and second order loops and presented as figures 9-2 and 9-3. The curves for a first order loop are used at low deviation ratios. The indicated subjective picture evaluations are made by 50 percent of viewers.[19] The optimum loop noise bandwidth is presented as a function of deviation ratio in Figure 9-4. Actually, the two-sided bandwidth, $2B_N$, is plotted.

The loop performance and design may be carried out with the aid of figures 9-2, 9-3, and 9-4, as follows. First, the desired output signal-to-noise ratio at threshold is found on figure 9-3 and the normalized threshold is determined. Since a safety margin will be added in any good communication system design, use of one radian is reasonable; however, at this threshold the loop will go out of lock fairly frequently. In figure 9-2, the required

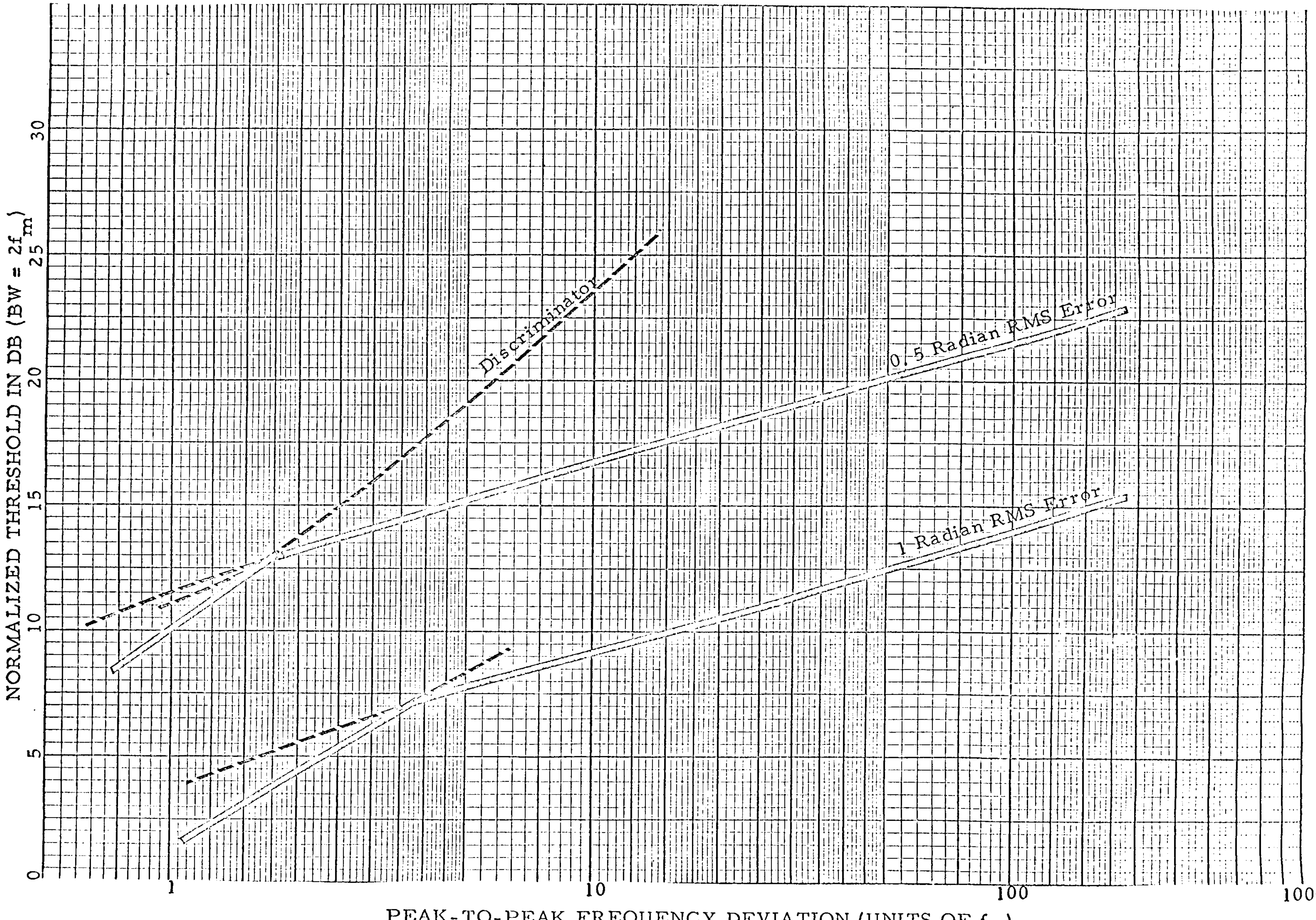

Figure 9-2. Phase Lock Loop Threshold

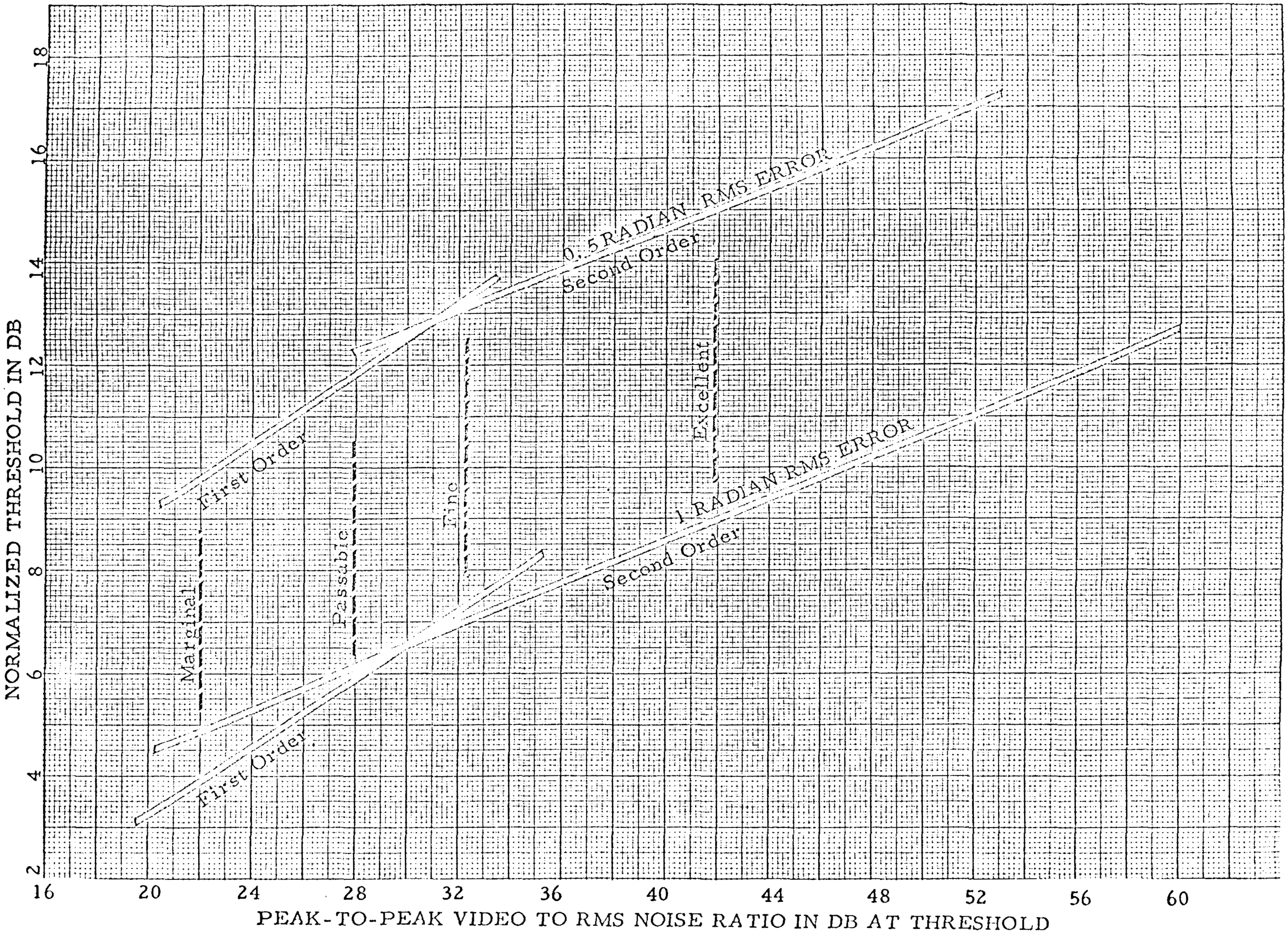

Figure 9-3. Phase Lock Loop Performance

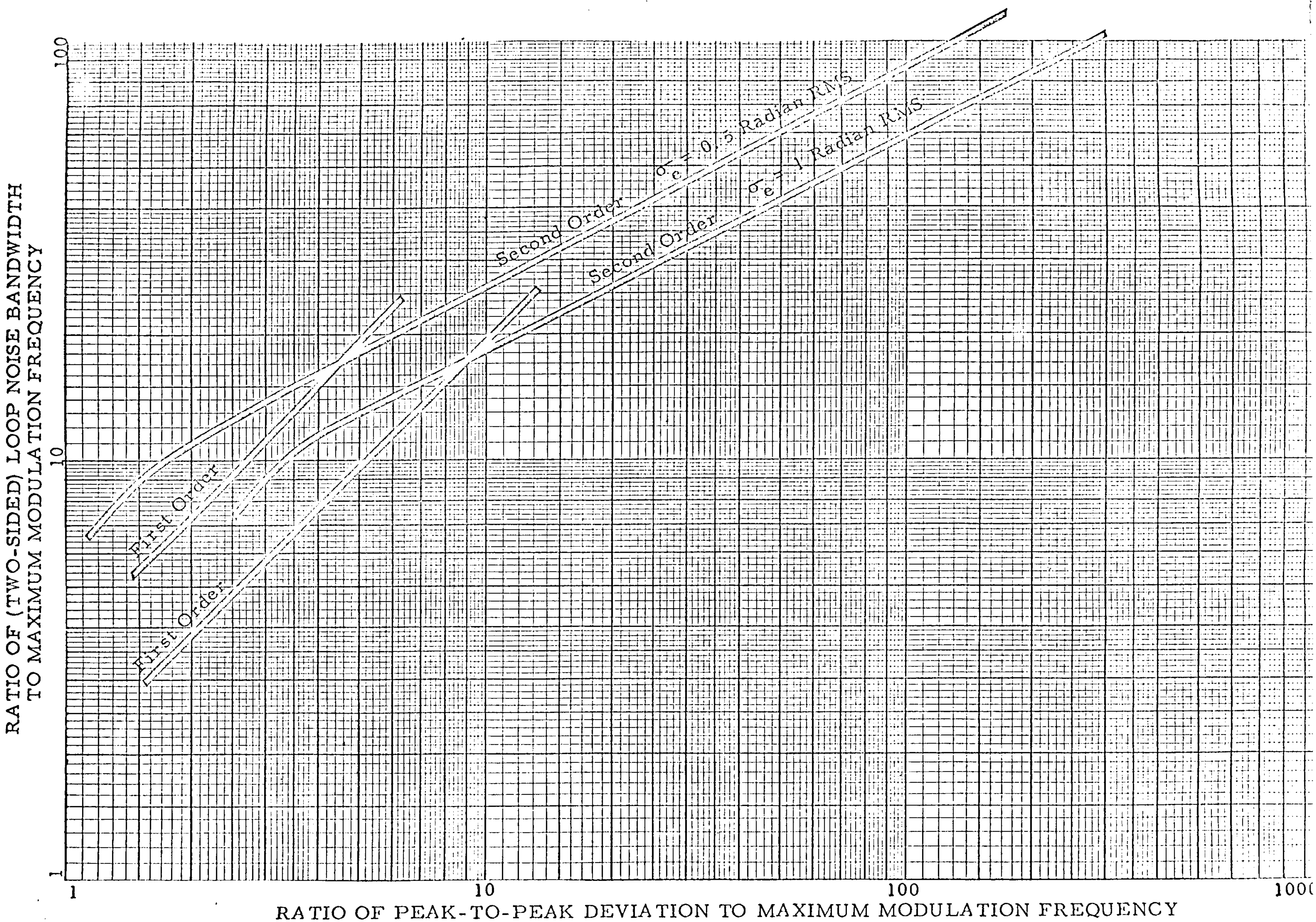

Figure 9-4. Optimum Loop Noise Bandwidth

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deviation ratio is found, and figure 9-3 yields the requisite loop noise bandwidth.

As an example, suppose that an output signal-to-noise ratio of 34 db is specified and f_m = 1mc. Then, for one radian error

Normalized threshold	= 7.4 db (2 mc bandwidth)
Peak-to-peak frequency deviation	= 8 mc
Two-sided loop noise bandwidth	= 16.5 mc

It will be noted that the two-sided loop bandwidth is 65 percent wider than the minimum required IF bandwidth of 10 mc. This means that a small reduction in threshold may be realized by restricting the predetection bandwidth to the 10-mc minimum.

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10. MULTIPLEXING OF ANALOG TV, TELEMETRY DATA, AND VOICE

Because of the large bandwidth disparity between the TV and the telemetry or voice, multiplexing all three on the same RF carrier will require a relatively small power increase, compared with that for TV alone. The signal characteristics are assumed to be

TV - 1 mc baseband, 34 db peak signal-to-rms-noise ratio (25 db average S/N for sine wave),

Telemetry - 100 kilobit/sec rate, 10^{-6} error rate,

Voice - 3 kc bandwidth, 30 db signal-to-noise ratio (for sine wave),

as described in section 1.

In opposition to the problem with digital TV, where multiplexing means bit interleaving, more multiplexing possibilities are open with analog TV. In particular, either frequency-division multiplexing (FDM) or time-division multiplexing (TDM) may be used.

10.1 FREQUENCY DIVISION MULTIPLEXING

From an equipment point of view, the simplest method of multiplexing is to place the voice and telemetry on subcarriers above the video baseband; the composite signal frequency modulates the carrier. Then, demultiplexing may be acomplished simply by filtering at the receiver. It will be noted that the phase lock loop discriminator has a noise bandwidth considerably greater than the video baseband; hence, the only significant problem with the subcarriers is in regard to modulation phase error produced in the loop.

The most efficient binary modulation for data transmission is biphase, and this is presumed for the telemetry subcarrier. The modulation error will be essentially the same as that produced by the unmodulated subcarrier, in view of the wide loop noise bandwidth. To yield the desired signal-to-noise ratio for voice, FM of the subcarrier is indicated. The deviation ratio will be selected so that the voice subcarrier threshold is not the limit on system performance. Again, the modulation error is essentially that of the unmodulated subcarrier.

Because of the narrow bandwidth of voice, its subcarrier will be given a higher frequency than the telemetry's, in view of the parabolic output noise

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spectrum from the phase lock loop. A practical frequency assignment should take into account cross-talk due to system nonlinearity and filter sharpness; however, the present discussion will presume close packing to yield theoretical system capability. Then, we may write

f_m = TV baseband cutoff = 1 mc

f_1 = Telemetry subcarrier frequency

f_d = Telemetry data rate = 100 kilobit/sec

f_2 = Voice subcarrier frequency

Δf = Voice frequency deviation

f_a = Audio bandwidth = 3 kc

The presumed relations between these frequencies are

$$
\begin{aligned}
f_1 &= f_m + f_d = 1.1 \text{ mc} \\
f_2 &= f_1 + f_d + \Delta f + f_a = 1.203 + \Delta f
\end{aligned}
\qquad (10\text{-}1)
$$

That is, the telemetry bandwidth is taken to be twice the bit rate, a pessimistic assumption, and the voice subcarrier bandwidth is equal to $2(\Delta f + f_a)$.

Measured in terms of instantaneous frequency deviation of the carrier, the telemetry subcarrier peak amplitude is ΔF_1, and the voice subcarrier amplitude is ΔF_2. The peak deviation produced by the TV signal is ΔF. It is desired to choose these deviations so that the loop carrier threshold coincides with the minimum subchannel requirements. Correspondingly, loop threshold will be computed on the basis of mean square modulation error.

The voice frequency peak deviation ratio $\Delta f/f_a$ is obtained immediately from figures 7-1 and 7-2 to be $5\sqrt{2}$, and the subcarrier threshold is 8.4 db referred to the bandwidth $2f_a$. Then, the voice subcarrier peak deviation is 21 kc, and $f_2 = 1.224$ mc. Also,

$$
\frac{2\pi^2 \Delta F_2^2}{4\pi^2 f_2^2 \, (2N_o f_a/S)} = 6.9 \qquad (10\text{-}2)
$$

The denominator of equation 10-2 is the phase lock loop output power (above threshold) in the vicinity of f_2 and measured in the bandwidth $2f_a$.

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Similarly, the telemetry threshold for 10^{-6} error rate in 10.6 db, referred to the bandwidth f_d, or

$$\frac{2\pi^2 \Delta F_1^{\ 2}}{4\pi^2 f_1^{\ 2} \ (N_o f_d / S)} = 11.4 \tag{10-3}$$

Finally, the TV threshold is 25 db, rms, so that

$$3\left(\frac{\Delta F}{f_m}\right)^2 \quad \frac{S}{2N_o f_m} = 316 \tag{10-4}$$

Equations 10-2, 10-3, and 10-4 relate the subchannel thresholds to the carrier threshold.

The mean square phase error remains to be found. Adding the various modulation error terms gives

$$\sigma_s^{\ 2} = \frac{1}{2}\left(\frac{\Delta F}{f_m}\right)^2 \frac{f_m^{\ 4}}{f_m^{\ 4} + B_o^{\ 4}} + \frac{1}{2}\left(\frac{\Delta F_1}{f_1}\right)^2 \frac{f_1^{\ 4}}{f_1^{\ 4} + B_o^{4}} + \frac{1}{2}\left(\frac{\Delta F_2}{f_2}\right)^2 \frac{f_2^{\ 4}}{f_2^{\ 4} + B_o^{\ 4}} \tag{10-5}$$

where $B_o = \sqrt{K}/2\pi$ is the loop resonant frequency. The deviation ratios may be substituted into equation 10-5 from equations 10-2, 10-3, and 10-4. Adding the mean square phase noise and making the approximation $B_o >> f_2$ yields

$$\frac{N_o}{S\ B_o^{\ 4}} \left(105\, f_m^{\ 5} + 11.4\, f_d f_1^{\ 4} + 13.8\, f_a f_2^{\ 4}\right) + 3.33 \frac{N_o}{S}\ B_o = \sigma_e^{\ 2} \tag{10-6}$$

B_o may be chosen to minimize S/N_o, and substituting for the various frequencies gives

$$B_o = 2.76 \text{ mc}$$

and for σ_e = one radian

$$\frac{S}{2N_o f_m} = 5.5 \quad (7.4 \text{ db}) \tag{10-7}$$

[redacted]

It is interesting to note that dropping the subcarriers completely yields a negligible reduction in threshold, according to equation 10-6 or figure 7-2.

The carrier deviation produced by each channel may now be computed from equations 10-2, 10-3 and 10-4. The results are

$$\begin{aligned} \Delta F_2 &= 106 \text{ kc} && \text{(voice)} \\ \Delta F_1 &= 500 \text{ kc} && \text{(telemetry)} \\ \Delta F &= 4.4 \text{ mc} && \text{(TV)} \end{aligned} \qquad (10\text{-}8)$$

The small deviations produced by the voice and telemetry subcarrier explain why there is such a small penalty associated with their introduction.

10.2 TIME DIVISION MULTIPLEXING OF TV, TELEMETRY DATA, AND VOICE

Although frequency division multiplexing of TV, telemetry, and voice appears to require the most simple equipment, crosstalk is a potential problem area in a FDM system. The phase detector nonlinearity will become particularly apparent for a phase error exceeding about 30°. For this reason, despite its somewhat greater complexity, time-division multiplexing may be considered to completely eliminate crosstalk. Furthermore, by taking advantage of the flyback time between horizontal lines, time-division multiplexing need introduce little, if any, transmitter power penalty.

To begin with, we note that the horizontal sweep rate of 7500 cps is quite adequate for sampling a 3-kc audio channel. By storing 14 telemetry bits for transmission during the flyback time, a telemetry data rate of 105 kilobits/sec is achieved; this is slightly more than the 100 kilobit objective.

Since the TV modulation is designed to yield a 34 db peak signal to rms noise ratio, the voice sample may be transmitted directly as a pulse whose amplitude deviates the carrier to a frequency between the peak limits. The voice quality will be acceptable. Now, the basebandwidth of 1 mc corresponds to a theoretical maximum pulse rate of 2×10^6 pulses/sec; and a rate of 10^6 pulses/sec is certainly usable. The time devoted to telemetry and voice is that necessary to send 16 pulses, including one for horizontal sync, or a total of 16 microseconds. This is only 12 percent of the line duration and, in contrast, commercial TV standards allow 18 percent of the line to be blanked.

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The telemetry data is transmitted by direct frequency modulation of the carrier, and the high output signal-to-noise ratio from the demodulator insures essentially a zero error rate. Because the telemetry pulses vary between minimum and maximum amplitude, the TV picture is not automatically blanked. For this reason, timing must be derived in the ground receiver to generate a synthetic blanking signal.

It will be noted that if the horizontal sync pulse precedes the telemetry and voice pulses and immediately follows the end of the line, it will serve to maintain timing synchronization. The problem remains of distinguishing the horizontal sync pulses from the rest of the composite signal, noting that a sync pulse above reference black would undesirably penalize system performance. In addition, frame sync must be derived.

By synchronizing the telemetry with the television, these problems can be resolved most simply. That is, the telemetry will contain frame synch information which, prior to the start of TV transmission, specifies where the TV frames will begin. In addition, the telemetry will specify the location of the horizontal synch pulses at least as accurately as the bit duration, or 10 μsec. The accuracy can probably be as good as 2 μsec, since the telemetry data rate will be accurately known. Taking the pessimistic value, however, the horizontal sync pulse must be unique within the 10 μsec uncertainty. Thus, the total required time interval for the horizontal sync, the voice pulse, and 14 bits of telemetry is, at most, 25 μsec, which is 18.7 percent of line duration. This approximately conforms to commercial standards.

A block diagram showing how time division multiplexing and demultiplexing can be implemented is presented as figure 10-1. This method of multiplexing is reasonably simple and is recommended over frequency division multiplexing because of the elimination of the crosstalk problem. A programmer provides the requisite timing to control gates connecting the information sources with the modulator. The vidicon output is cutoff during flyback, and the voice waveform is simply sampled once to obtain a PAM sample. The 13-bit shift register is required to accumulate telemetry bits arriving at a uniform approximate 100 kilobit/sec rate prior to telemetry transmission. The register is read out at a nominal one megabit/sec rate.

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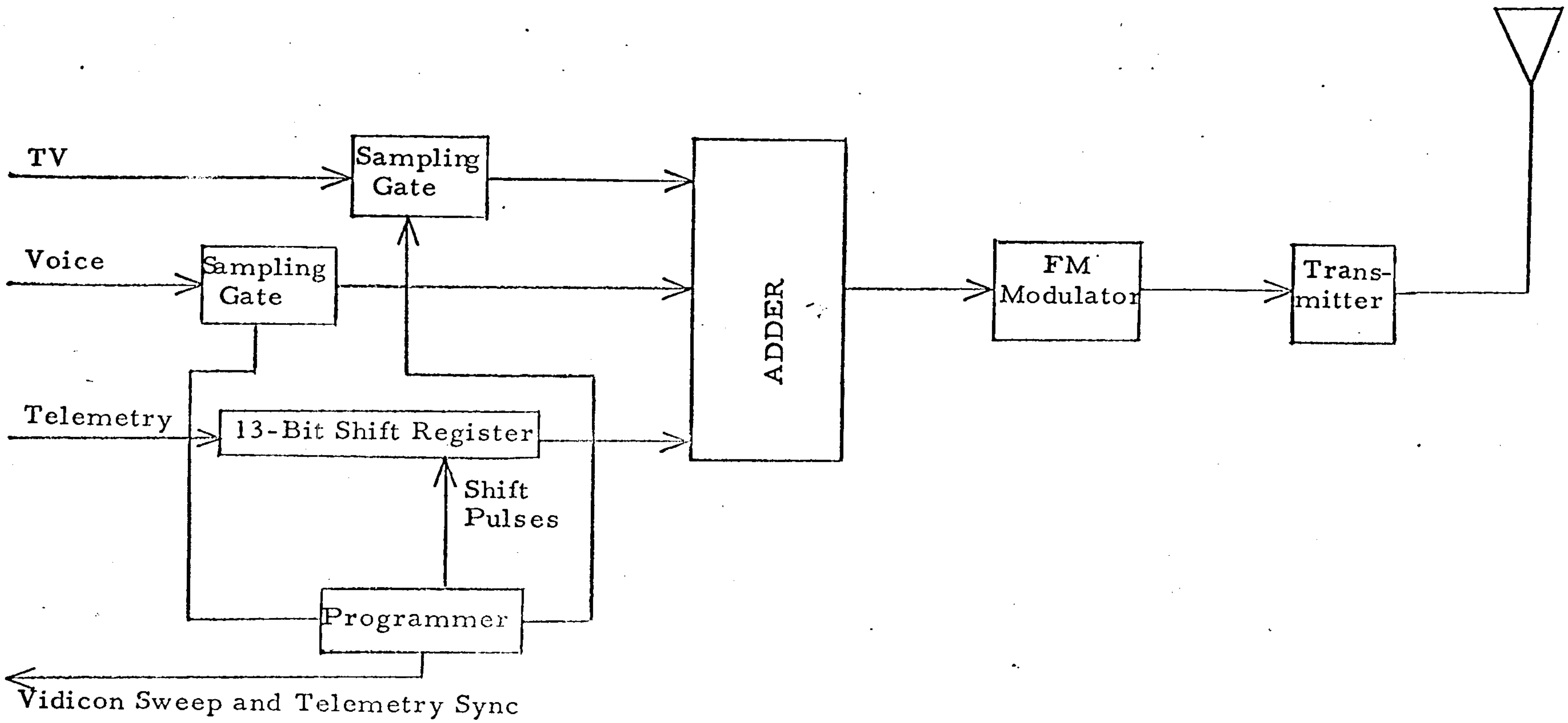

Figure 10-1. Time Division Multiplexer

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No additional buffer storage is necessary, since the 14^{th} telemetry bit may be assumed to arrive just after the previous 13 have been transmitted.

The vidicon sweeps and the telemetry must be synchronized with the programmer for the scheme outlined. It may be possible to eliminate the shift register by non-uniform commutation of the telemetry data. That is, the telemetry bits are stored in the telemetry subsystem and read out as required for transmission.

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